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PREGLED MODELA SIVE KUTIJE ZA MODELOVANJE I OPTIMIZACIJU HVAC&R SISTEMA

Key words: white-box modelling; black-box modelling; grey-box modelling; HVAC&R

REVIEW OF GREY-BOX MODELS FOR MODELING AND OPTIMIZATION OF HVAC&R SYSTEMS

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Model „sive kutije" kombinuje pristupe zasnovane na fundamentalnim fizičkim principima, poznate kao modeli „bele kutije", sa modelima zasnovanim na eksperimentalnim podacima, odnosno modelima „crne kutije", sa ciljem kreiranja modela koji predstavlja kompromis i omogućava precizno opisivanje složene dinamike sistema. Ovaj hibridni pristup posebno je pogodan u domenu KGH sistema, gde izražene termodinamičke nelinearnosti, nepredvidivi radni uslovi i nepotpuno poznavanje karakteristika procesa ograničavaju primenljivost analitičkih modela, dok primena empirijskih pristupa u promenjenim uslovima zahteva veliku količinu, najčešće merenih podataka. Tokom protekle decenije, modeli 'sive kutije' uspešno su primenjeni na širok spektar izazova u KGH sistemima, uključujući dinamičko modelovanje i predikciju termičkog ponašanja posmatranog objekta, optimizaciju potrošnje energije, detekciju i dijagnostiku grešaka, kao i razvoj upravljačkih modela koji integrišu veštačke neuronske mreže sa diferencijalnim jednačinama. U radu će biti razmatrane osnovne karakteristike kombinovanog modela sive kutije, kao i aktuelne inženjerske primene relevantne za modelovanje, kontrolu i optimizaciju savremenih KGH sistema, uz pružanje preporuka za buduće istraživačke pravce.

The grey-box model combines approaches based on fundamental physical principles, known as white-box models, with data-driven, known as black-box models, to create a model that represents a compromise and enables an accurate representation of complex system dynamics. This hybrid approach is particularly well-suited to the domain of HVAC&R systems, where pronounced thermodynamic nonlinearities, unpredictable operating conditions, and incomplete knowledge of process characteristics limit the applicability of purely analytical models. In contrast, the use of empirical approaches under varying conditions demands a significant amount of typically measured data. Over the past decade, grey-box models have been successfully applied to a wide range of challenges in HVAC&R systems, including dynamic modeling and prediction of thermal behavior of the observed system, optimization of energy consumption, fault detection and diagnosis, as well as the development of control models that integrate artificial neural networks with differential equations. This paper

highlights the fundamental characteristics of grey-box models, along with current engineering applications relevant to the modeling, control and optimization of modern HVAC&R systems, and provides recommendations for future research directions.

1. Introduction

Improving indoor air quality, maintaining thermal comfort, and ensuring effective refrigeration performance represent fundamental objectives of heating, ventilation, air-conditioning, and refrigeration (HVAC&R) systems across buildings of diverse functional typologies. Effective modeling of energy consumption in such systems has become one of the central research domains in contemporary thermal engineering over the past decade. Determining the overall energy demand in buildings is an inherently complex task due to the large number of interdependent factors, many of which are highly variable and difficult to quantify with precision. As a result, energy-demand, as well as energy use prediction, has evolved into an inherently interdisciplinary research area that integrates thermal engineering, energy systems, building physics, applied mathematics, and computational science.

Within the scope of modern research, three principal modeling paradigms have been established: white-box, black-box, and grey-box models. Each paradigm is rooted in a distinct theoretical foundation and characterized by different assumptions regarding system knowledge, data requirements, and the level of physical abstraction.

The first category, referred to as white-box models, is grounded in the application of fundamental physical laws and the numerical solution of heat-transfer differential equations. These models provide physically faithful representations of thermal and fluid processes occurring within HVAC&R system components and require extensive knowledge of building physics, thermodynamic properties of the working fluid, boundary conditions, and operational regimes. To support such analyses, numerous specialized simulation environments such as DOE-2, EnergyPlus, TRNSYS, BLAST, ESP-r, HAP, and IES APACHE have been developed to enable detailed evaluation of thermal and energy flows in buildings [1]. For example, a recent study used TRNSYS 18 to simulate a hotel building with 100 guest rooms and fan-coil HVAC systems. The model was calibrated using measured indoor temperatures taken every five minutes, adjusting infiltration, HVAC equipment parameters, and control logic. After calibration, the model predicted room temperatures with a root mean square error (RMSE) of 0.79 °C and a variation of 3.6 percent, well within international guideline limits. These results show that white-box simulation tools, when carefully calibrated with real operational data, can accurately represent HVAC performance and indoor thermal behavior across multiple zones [2]. A comparative study at the experimental test facility FLEXLAB evaluated EnergyPlus, DOE-2.1e, and DOE-2.2 for conventional commercial buildings with overhead mixing ventilation, finding that all three simulation engines predict cooling and heating loads with similar accuracy, typically within ±1.5% bias and around 10% root mean square error, demonstrating comparable reliability for HVAC performance modeling [3].

In contrast, black-box models belong to the class of data-driven methods that establish mathematical relationships between input and output variables based on experimentally acquired or operational data. These models commonly employ statistical and machine-learning approaches, such as regression analysis, optimization algorithms, and artificial neural networks. While they lack interpretability due to the absence of explicit physical representation, black-box models often achieve high predictive accuracy and are particularly advantageous when the system exhibits complex, partially known, or difficult-to-measure physical behavior.

Grey-box models represent an integrative modeling framework that synthesizes the complementary strengths of the white-box and black-box paradigms. The structural form of a grey-box model is derived from fundamental physical principles, ensuring interpretability and physical coherence, whereas parameter values are identified using advanced estimation algorithms applied to experimental or operational data. This hybrid approach enables high predictive performance while maintaining robustness, stability, and generalizability across a wide range of operating conditions. Recent studies show that grey-box models can be both accurate and computationally efficient. For example, a water-based heating and cooling system model, validated against a detailed physical model and measurements, predicted return temperatures with very small deviations and room air temperatures with minimal error, while completing a full one-year simulation in just 18 seconds compared to 8,229 seconds for the detailed model [4]. Similarly, a non-linear grey-box model of supermarket refrigeration cabinets, calibrated with one-minute operational data over 2.5 years, successfully estimated physically meaningful parameters such as cabinet temperature and evaporator refrigerant mass. The model proved robust and accurate, enabling classification of cabinets as open or closed and demonstrating suitability for large-scale implementation, fault diagnostics, and predictive control [5].

The literature consistently shows that physically grounded (white-box) models offer strong generalization capabilities, yet their accuracy is often constrained by the simplifying assumptions required for practical implementation. In contrast, data-driven (black-box) models typically demonstrate high predictive precision within their training domain but lack robustness and exhibit limited transferability beyond it. Grey-box models therefore represent an optimal compromise between these two extremes: they combine the generalization capacity of physically based approaches with the adaptability and accuracy characteristic of data-driven methods. Owing to this dual nature, grey-box models have gained widespread application in the optimization of HVAC&R system operation, the modeling and analysis of dynamic building processes, and the development of advanced control strategies that integrate physical knowledge with operational data. Within the realm of physically based modeling, an HVAC&R system is typically decomposed into interconnected subsystems in order to reduce overall modeling complexity. This decomposition enables independent representation and analysis of key components such as heat exchangers, steam humidifiers, temperature and humidity sensors, pumps, air-handling units, and other elements critical to system operation.

2. Structured Overview of Grey-Box Modelling Approaches

Grey-box models represent a methodological middle ground between physics-based approaches and fully data-driven methods, integrating established physical principles with empirical information to yield models that are both interpretable and sufficiently flexible. In their broadest definition, grey-box models constitute hybrid constructs that combine physics-based models, numerical simulation frameworks, and statistical or machine-learning models.

In general, grey-box models can be classified according to the manner in which physical and empirical components are integrated, the mathematical structure employed, and the identification strategy used. Consequently, grey-box models have been categorized along multiple methodological dimensions, including discrete versus continuous representations, static versus dynamic formulations, linear versus nonlinear structures, deterministic versus probabilistic frameworks, and explicit versus implicit configurations [6].

In typical development workflows, the foundational structure of a grey-box model is inherited from first-principles knowledge and subsequently refined using experimental data. The modelling

process generally involves three core steps: (1) formulation of a simplified physics-based representation of the system; (2) assignment of physical parameters derived from thermodynamic, heat-transfer, or fluid-mechanics relations; and (3) parameter identification using operational data to refine or compensate for modelling uncertainties. The relationship between these three model types and the underlying knowledge sources is shown in Figure 1 [7].

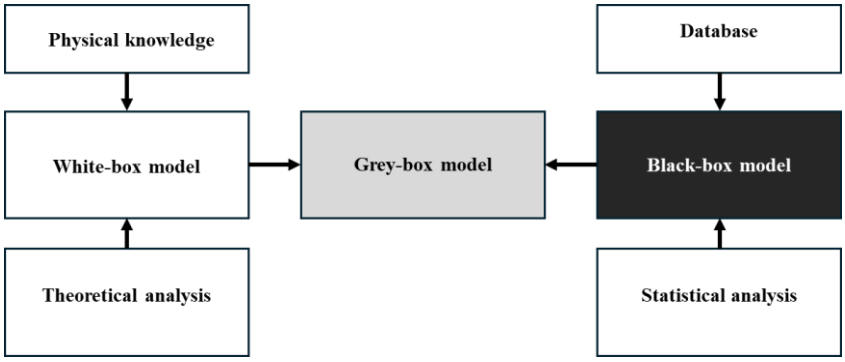

Figure 1. Relationships among physics-based white-box, statistics-based black-box, hybrid grey-box models and knowledge sources [7]

Grey-box models are most commonly categorized into serial and parallel hybridization approaches [8].

The serial approach sequentially bridges the gap between theoretical knowledge and experimental observations [9]. In such configurations, uncertainties or structural deficiencies associated with an incomplete white-box model are reduced in a second stage through empirical corrections derived from measured data. A grey-box model with a serial structure therefore focuses on minimizing the discrepancy between physical-model predictions and the actual measured response. For instance, Duarte et al. [10] proposed a hybrid methodology that integrates physics-based models with, rather than relying solely on, statistical residual models to enhance predictive accuracy. In this framework, the primary model is derived from first-principles system behavior, while a secondary model captures the residual discrepancies between measured data and analytical predictions. Recent advancements have extended this concept through physics-informed machine learning, incorporating conservation laws, stability constraints, or thermodynamic inequalities directly into data-driven model training. Krannichfeldt et al. [11] systematically compared four hybrid strategies: residual, surrogate, augmentation, and others, alongside purely physics-based and data-driven models across real-world buildings. The study shows that the residual approach, a type of serial hybrid method, was the most effective for predicting indoor temperatures, achieving high accuracy while maintaining the physical coherence of the primary model. The surrogate approach, in contrast, consistently exhibited the lowest accuracy [11].

In contrast with serial approaches, in which empirical corrections follow the physical model, parallel grey-box models process input data through both physical and empirical components at the same time, integrating outputs to capture intricate system behaviors. This allows estimation of responses that would otherwise be difficult to predict, given incomplete system knowledge. This modeling approach combines available prior knowledge with an artificial neural network (ANN) to derive an estimator of unmeasured process parameters. Such hybrid estimators frequently rely on ANNs or other nonlinear regressors, which can interpolate and extrapolate far more accurately than standalone black-box models while requiring significantly fewer training samples due to the presence of the physics-based backbone.

Mathematically, grey-box models may adopt linear or nonlinear formulations, deterministic or stochastic dynamics, and can be expressed as ordinary differential equations (ODEs), differential algebraic equations (DAEs), or reduced-order partial differential equations (PDEs) systems, depending on the desired spatial and temporal resolution, depending on the application requirements. Linear deterministic formulations dominate supervisory HVAC&R control and building-energy simulation due to their computational tractability, whereas nonlinear or stochastic grey-box models are applied in scenarios characterized by strong boundary-condition variability, measurement noise, or inherently nonlinear thermodynamic behavior [12], [7].

Among grey-box methodologies, the resistance–capacitance (RC) model is frequently applied in HVAC&R systems, combining thermal resistances (R) to represent heat-transfer paths with thermal capacitances (C) representing the heat-storage capacity of building structures or HVAC&R components. Numerous studies have focused on determining the ideal number of R and C elements in such models. A widely reported recommendation is that two capacitances are often sufficient, as increasing the number of capacitances beyond this level yields diminishing improvements in accuracy. These simplified physical structures contain combined parameters, such as lumped thermal resistances and heat capacities, which can be estimated through system identification procedures using operational data [4]. Reynders et al. [13] developed reduced-order grey-box models using data-driven identification to address unknown thermal properties, evaluating their predictive accuracy and parameter interpretability using simulation datasets.

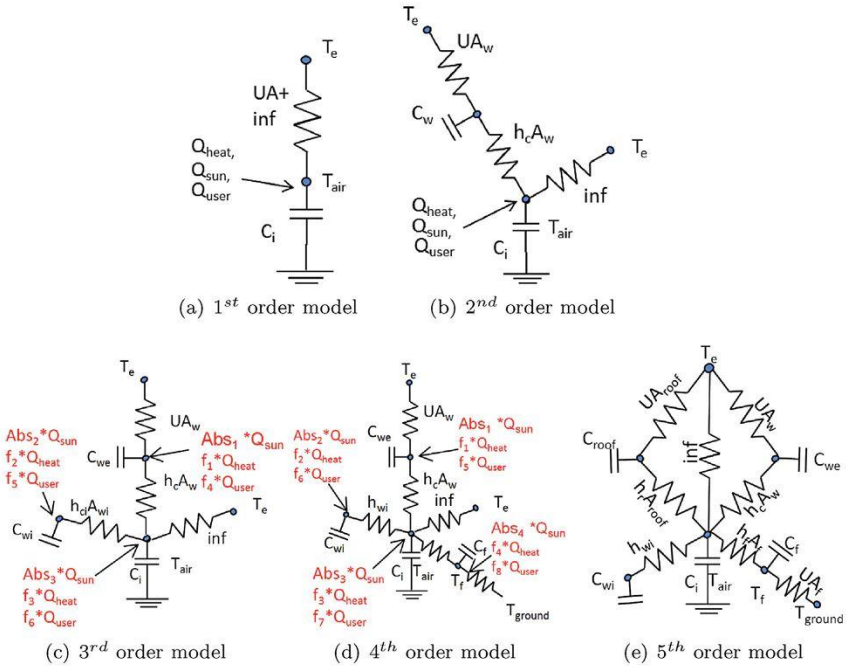

Figure 2. Resistance–capacitance (RC) analogy of the reduced-order building model introduced by Reynders et al. [13]

In several recent experimental studies, low-order RC grey-box models have demonstrated strong prediction accuracy when calibrated with measured data. For example, a 2023 study developed a six-resistor two-capacitance (6R2C) model to predict indoor air temperature under extreme climate conditions [14]. The results show that including the influence of cladding significantly improves predictions, reducing the maximum absolute temperature error from 5 °C to 1 °C [14]. Another study applied a 2R2C grey-box model combined with automated parameter identification to a naturally ventilated nearly-zero-energy building (nZEB), reducing calibration effort by approximately 90 %

while keeping the temperature prediction root mean square error (RMSE) around 0.4 °C, showing that RC-based grey-box models remain effective even under challenging ventilation and environmental variability [15].

Linear grey-box models appear extensively in the literature ([16], [17]), typically assuming linear relationships between input variables and system responses. Yanfei et al. [18] reviewed existing grey box models and found that relatively few studies employed the "Forward Approach," wherein R and C parameters are derived directly from physical relations. Instead, most investigations rely on an "Inverse Method," in which the structure is physically motivated, but parameters are identified from measured data. Thilker et al. [19] developed a nonlinear grey-box model based on stochastic differential equations using an inverse identification approach. Their model, which integrated a hydronic radiator heating system, accurately predicted indoor air temperature, return temperature, and heat output in an unoccupied school building over a week, and demonstrated that solar radiation exerted a limited influence. However, a key limitation of their method is the need for large training datasets to ensure parameter identifiability.

Harb et al. [20] examined a grey-box identification strategy for forecasting building thermal behavior and observed that an R4C2 structure provides the highest accuracy for indoor climate prediction. In their formulation, convective heat transfer between the air and enclosing surfaces is linearized, and thermophysical properties are assumed uniform across building elements. Berthou et al. [21] evaluated four grey-box structures without water-based heat-emission systems, concluding that an R6C2 model is necessary when thermal loads are high. They further recommended monthly recalibration of model parameters, which requires separate identification procedures for each month.

In summary, grey-box models can be rigorously classified according to their structural architecture, mathematical formulation, identification methodology, and degree of physical prior knowledge. This systematic classification clarifies methodological distinctions and highlights key scientific advances over the last two decades. Structurally, reduced-order RC networks and lumped-parameter dynamic models remain dominant due to their balance between physical interpretability and computational tractability, particularly in building thermal dynamics, refrigeration equipment, and thermal storage systems. Mathematically, grey-box formulations have evolved from linear time-invariant state-space models to nonlinear, stochastic, and hybrid approaches, enabling the representation of complex thermodynamic phenomena such as phase change, internal gains, and variable heat-transfer coefficients. Finally, the integration of physical prior knowledge via conservation laws, thermodynamic constraints, or geometry-based parameterization, distinguishes grey-box models from purely data-driven methods. This combination of physics and data has enhanced model generalizability, explainability, and control readiness, supporting advanced applications such as model predictive control, fault detection and diagnostics, and energy flexibility management.

3. Applications of Grey-Box Models in Practice

Grey-box models are primarily used to predict the dynamic thermal behavior of buildings, capturing indoor temperature responses to varying external conditions and system operation. They serve as a foundation for Model Predictive Control (MPC), enabling optimization of heating and cooling schedules to maintain thermal comfort while minimizing energy consumption. Additionally, grey-box models are applied in system identification and parameter calibration, allowing model parameters to be adjusted using measured data for improved accuracy. Finally, they support energy consumption assessment at the zone or building level, facilitating energy efficiency analysis and informed operational decision-making.

To provide a comprehensive perspective on the versatility and practical applications of grey-box models in HVAC&R systems, the following section reviews selected studies in which authors have implemented grey-box modeling across different building types, system configurations, and operational objectives. These references illustrate how grey-box models have been applied to commercial and residential HVAC&R systems, integrated heating and cooling networks, and advanced energy simulation frameworks, highlighting both their predictive capabilities and adaptability for energy optimization, control, and fault detection. By examining these examples, the aim is to clarify the strengths, limitations, and practical considerations associated with grey-box modeling, setting the stage for a deeper discussion of their role in modern building energy management.

The authors in [22] presented a gray-box modeling methodology for HVAC&R systems in commercial buildings that integrates simplified physical representations with data-driven parameter estimation in order to achieve an accurate, computationally efficient, and practically adaptable simulation framework. This approach was developed to address the inherent limitations of purely black-box models, which, although easy to implement and fast in computation, often fail to capture the true physical behavior of building systems and exhibit poor reliability when operating conditions fall outside the training dataset or when measurement data are incomplete. Additionally, the authors noted that conventional gray-box models frequently require extensive manual parameter tuning, making them complex and highly dependent on the expertise and subjective judgment of the modeler. To overcome these challenges, the authors developed an automated parameter fitting mechanism in which time-varying equipment-level parameters are continuously estimated using a multilayer perceptron neural network with ReLU (Rectified Linear Unit) activation. These parameters were constrained within physically meaningful ranges and optimized to ensure stability and prevent overfitting. By minimizing the number of parameters and enabling their real-time adjustment, the proposed methodology significantly reduces manual intervention while preserving the physical interpretability of the model. The framework relies only on essential measurable variables, such as temperatures and flow rates, allowing the model to replicate system behavior and detect abnormal operation without the need for detailed design information or computationally intensive high-fidelity simulations. The methodology was validated using a simplified HVAC&R system implemented in TRNSYS to generate virtual operational data under controlled conditions. Each device was modeled separately based on governing physical equations, and its parameters were trained using machine learning techniques. Upon validation of individual device performance, the approach was extended to simulate the entire system, enabling the analysis of component interactions and system-level dynamics. The results indicated high predictive accuracy for pumps and heat exchangers.

At the system level, the model demonstrated reliable performance in predicting chilled water outlet temperatures, COP values, and overall power consumption, though minor deviations were caused by simplified flow modeling and the absence of heat loss representation in the heat storage tank. The total predicted system energy consumption over a three-month period exceeded the TRNSYS reference by approximately 14%, while the aggregated predictions of individual device models showed closer conformity [22]. These findings highlight both the robustness and current limitations of the approach, emphasizing the influence of parameter accuracy and subsystem representation on overall model fidelity. The authors demonstrated that the proposed gray-box modeling approach enables adaptive, real-time simulation of HVAC&R systems with reduced modeler workload and improved operational insight. By combining physical structure with machine learning-based parameter calibration, the method provides a balanced solution that is both interpretable and scalable, making it suitable for practical applications such as performance monitoring, energy optimization,

and fault detection in complex commercial building environments, while also identifying clear pathways for future refinement through enhanced modeling of control signals, flow dynamics, and thermal storage behavior.

Authors who have made very significant contributions in the field of grey-box modeling, Afram and Janabi-Sharifi [16], conducted a comprehensive experimental and analytical investigation of a complex residential HVAC&R system composed of several interconnected subsystems, including Energy Recovery Ventilation (ERV), an Air Handling Unit (AHU), a buffer tank (BT), radiant floor heating (RFH), and a ground source heat pump (GSHP). Since the raw dataset contained noise, missing values, and redundant information unsuitable for system identification, a structured pre-processing procedure was applied, including median filtering to suppress measurement noise and downsampling to retain essential dynamic trends while improving computational efficiency. Based on the processed data, multiple model structures were developed and compared, including black-box approaches such as ANNs, Auto-Regressive models with eXogenous inputs (ARX), transfer function models (TF), state-space models, and process models, alongside gray-box models previously developed using simplified physical representations of the system. The primary objective of the study was to evaluate the predictive capability of these different modeling paradigms and to assess the suitability of gray-box models as an intermediate solution that combines physical insight with data-driven calibration. Model performance was evaluated through direct visual comparison of measured and simulated outputs as well as through analytical performance metrics, allowing a systematic ranking of each modeling approach across all subsystems. While ANN models achieved the highest overall accuracy, followed closely by ARX and TF models, the results revealed that gray-box models maintained consistent and physically meaningful predictions, particularly in subsystems characterized by complex thermal interactions and noisy or limited data, such as the radiant floor heating system. In these cases, the inclusion of physical system knowledge enabled gray-box models to better capture the true behavior of the system compared to purely data-driven models. Afram and Janabi-Sharifi [16] demonstrated that although black-box models can offer superior numerical accuracy under certain conditions, gray-box models provide a more robust, interpretable, and practically applicable framework for HVAC&R system modeling. Their capacity to balance predictive performance with physical consistency makes them especially valuable for applications such as energy optimization, fault detection, and model predictive control in real building environments, highlighting their strategic importance for the development of intelligent and efficient HVAC&R control systems.

Li et al. [18] conducted an analysis of RC models as one of the most commonly used grey-box modeling approaches and outlined both the main advantages and the key drawbacks associated with their application. The primary objective of their study was to provide a comprehensive understanding of the theoretical foundations, practical applications, and software tools related to grey-box modeling, while also identifying existing challenges and research gaps. The review addresses the theoretical basis and inherent limitations of this approach, with particular emphasis on the assumptions underlying RC modeling. It examines the modeling of building elements such as walls, thermal zones, and other envelope components, as well as the use of schematic representations and techniques for model order reduction. Two principal strategies for model development are discussed: the forward (bottom-up) approach, which is grounded in physical descriptions of building components, and the inverse (data-driven) approach, which relies on measured data for parameter identification. Together, these strategies enable a balance between physical transparency and adaptability to real operational data. Grey-box models are commonly used to analyze heat transfer through building envelopes, including

advanced constructions such as phase-change material walls and radiant floor systems, offering valuable insight into both conventional and innovative design solutions. In addition, RC models support accurate and computationally efficient estimation of building thermal loads, which is essential for effective energy management and control. The literature indicates a dominance of single-zone implementations, although multi-zone configurations are increasingly being adopted, with second-order models most frequently selected due to their favorable compromise between accuracy and computational simplicity. Reliable load estimation enabled by RC models underpins both primary and secondary control strategies. These models are particularly well-suited for supervisory control applications, including MPC, where they contribute to efficient temperature regulation and energy optimization. Their low computational burden also makes them suitable for large-scale applications, such as district or urban energy modeling, facilitating integrated planning and enhanced interaction between buildings and power grids. Implementation platforms range from general-purpose programming languages to specialized environments such as Modelica, with grey-box models increasingly supporting smart grid integration and advanced energy management strategies. Although numerous software packages are available for reduced-order modeling in environments such as MATLAB, Python, Modelica, R, and C++, most are either highly specialized for specific research purposes or demand significant technical expertise [18]. This reveals a clear need for a more accessible, general-purpose, and user-friendly platform to promote broader adoption of grey-box modeling approaches. Despite their recognized benefits, RC models face several limitations that hinder their widespread implementation. These include ambiguities in theoretical assumptions, inconsistent naming conventions and structural formulations (e.g., variations of the xRyC framework, in which x is the number of thermal resistances, and y is the number of thermal capacitances.), uncertainty in choosing between forward and inverse modeling techniques, challenges in determining the optimal model order, and their relatively narrow application focus beyond load and energy estimation. The absence of generic, intuitive software tools further contributes to these barriers.

Mörth et al. [4] presented a recent study, published in 2025, focusing on the development and validation of a non-linear grey-box model for building energy simulations, with particular emphasis on water-based heating and cooling systems. The proposed model was designed to achieve a high level of numerical efficiency and accuracy while relying on a minimal set of input data, making it especially suitable for applications such as urban-scale building energy modeling and MPC. The case study was conducted on an office building at Graz University of Technology, consisting of four floors and a total floor area of 2,480 m². The building is equipped with a concrete core activation system for heating and cooling, supplied by a local energy network through two heat exchangers. Unlike conventional control strategies, system operation is predominantly governed by outdoor air temperature rather than indoor room temperatures, reflecting real operational conditions. The authors developed a detailed white-box model using IDA ICE software, which included 38 thermal zones and was calibrated using both a winter heating experiment and annual measured energy consumption data. This high-fidelity model served as a benchmark for the formulation of the grey-box model, which was constructed as a 3R2C RC network and represented the entire building as a single thermal zone. The grey-box model incorporated key thermal capacities and resistances, as well as the effects of solar gains, infiltration, and air change rates. Two variants were tested: GB1, calibrated using parameters from the detailed white-box model, and GB2, calibrated directly with measured operational data. Sensitivity analysis using the Morris method [4] guided parameter selection and minimized overfitting. Validation showed that GB1 accurately reproduced the detailed model's thermal behavior, while GB2 demonstrated excellent agreement with real measurements, with energy demand deviations of

only 0.2% for cooling and 1.7% for heating. GB2 also achieved slightly more accurate indoor temperature predictions due to direct calibration with measured data. Limitations include the single-zone representation, which cannot fully capture multi-zone interactions and simultaneous heating/cooling demands, and minor inaccuracies in heat flow predictions due to simplified thermal mass modeling. Overall, the results highlight the effectiveness of grey-box models in balancing predictive accuracy and physical interpretability, confirming their value for energy-efficient building simulation and HVAC&R system optimization.

Table 1 contains a list of studies useful for understanding the topic of grey-box models and their applications, which were not used as main references in the paper, but provide practical examples, comparisons, and analyses on the subject.

Table 1 A review of additional useful studies on the application of grey-box models in HVAC&R systems

Source	Year	The Focus of Article (Objective)
Estrada-Flores et al. [23]	2006	The authors review the development and validation of grey-box models for refrigeration systems, emphasizing parameter-estimation techniques and practical implementation considerations.
Ji & Xu [24]	2015	A grey-box–based indirect submetering method is proposed to estimate HVAC end-use energy in office buildings, achieving accurate disaggregation without physical submeters.
Amara et al. [25]	2015	The study compares thermal modeling approaches for building energy management and identifies grey-box reduced-order models as an effective compromise between accuracy and computational effort.
De Coninck et al. [26]	2016	A toolbox for constructing and validating grey-box building models is presented, enabling automated parameter estimation and reliable forecasts for control applications.
Saurav [27]	2017	A grey-box thermal model is developed for buildings integrated in district heating and cooling networks, demonstrating dependable prediction accuracy for network-level control tasks.
Xiong & Wang [28]	2020	A semi-physical grey-box model is formulated to optimize HVAC energy consumption, showing strong potential for supporting energy-efficient operation.
Ahamed et al. [29]	2020	A grey-box virtual sensor is introduced to estimate AHU supply air temperature with high accuracy, reducing reliance on physical sensors and enabling improved monitoring and control.
Kathirgamanathan et al. [30]	2021	A comprehensive review highlights grey-box and data-driven predictive models as key tools for unlocking building energy flexibility and enabling grid-responsive operation.
Kim et al. [31]	2022	A review of HVAC MPC research identifies grey-box models as particularly valuable due to their balance of physical interpretability and computational tractability for real-time control.
Klanatsky et al. [32]	2023	A refined grey-box model tailored for building MPC is developed, achieving high predictive accuracy and robust parameter identification.
Seiler et al. [33]	2023	A systematic comparison of grey-box model structures for HVAC systems is conducted, illustrating how model architecture influences predictive performance and suitability for control.
Lin et al. [34]	2024	A grey-box model capable of inferring unmeasured internal heat gains is developed, enhancing prediction accuracy in buildings with variable occupancy and dynamic internal loads.
Tugores et al. [35]	2025	A hybrid grey-box model is formulated to jointly capture indoor air quality and thermal dynamics, enabling integrated analysis of comfort, ventilation, and energy performance.

4. Conclusion

Grey-box modeling has emerged as a highly effective approach for simulating and optimizing HVAC&R systems, offering a balanced compromise between the physical interpretability of white-box models and the predictive accuracy of black-box models. This paper reviewed the classification, and practical applications of grey-box models, highlighting their methodological flexibility, ability to integrate empirical data, and adaptability across diverse building types and system configurations.

The analysis demonstrates that grey-box models, particularly RC-network-based frameworks, provide robust and physically meaningful predictions even under conditions of limited or noisy data. They support critical applications such as energy consumption assessment, MPC, system identification, and fault detection, while reducing manual tuning efforts through automated parameter calibration. Case studies from recent literature, including both commercial and residential buildings, underscore the practical advantages of grey-box modeling in capturing system dynamics, optimizing energy performance, and facilitating real-time operational decision-making.

Despite their strengths, current grey-box approaches face limitations, including challenges in modeling multi-zone interactions, thermal inertia, and complex flow dynamics, as well as inconsistencies in model structures and software accessibility. Future research should focus on extending these models to multi-zone and district-scale systems, enhancing parameter identification methods, and developing more user-friendly, general-purpose modeling tools to promote broader adoption.

Grey-box models represent a strategically important methodology for advancing energy-efficient building design and intelligent HVAC&R control, bridging the gap between theoretical rigor and practical applicability in modern building energy management.

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