OPTMIZATION OF POWER PLANT OPERATION

Lj. Janjusevic[1], P. Hadzic[1], M. Radosavljevic[1]

[1] GOSA Institute, 25 Milana Rakica, Belgrade, Serbia

E-mail: **ljubinkoj@yahoo.com**

Abstract: The paper discusses the method of optimization and particularities of a plant in the field of energy technologies. Also the proposals and comparisons are made of the possible modes of a system in order to maximize its efficiency. The problem is solved by using the dynamic programming method, which is based on the principle of optimality.
It is estimated that up to 40% of an energy system's costs cover the way it is organized and utilized (or not). It certainly is one of the areas where costs can be reduced. Utilization of "hidden" reserves largely depends on the adaptability of the environment with which the system communicates, it becoming increasingly prominent. Optimization of technological characteristics of a system provides the relationship of various physical quantities and participates in maximizing its efficiency. Based on that, in the field of process engineering extensive research has been carried out on the power plant utilization, while noting that the design is the phase in which the least was done.

Key words: power plant, optimization, working fluid.

1. INTRODUCTION

The main issue in power installations is how to define processes at low cost and in a manner to satisfy the diverse requirements of users. Therefore, modern enterprises have to become as responsive as possible to the demands of customers, in order to satisfy them. Forming a flexible cell represents one step in the direction of adaptability and rapid response to customer requirements. Especially the ability to adapt themselves to changes in demand proved to be a key feature of flexible plants. Flexible systems are renowned for their ability to work efficiently in all conditions and quickly respond to changes in the combination of various requirements. Therefore their development is among the most important events in the industrial automation.
"Defining operations" refers to the allocation of tasks to individual machines in the system, according to the sequence of operations prescribed by the process technology, which are subject to plant restrictions. Clearly, this planning decision is as much in connection with the operations of the technological process in the system, as measures for the movement of materials must be undertaken expressly. If this is not taken into account, planning this task can lead to low utilization of machines and other equipment, and a longer production cycle. Thus, power plants provide one of the main unifying functions in the system.
Optimization tasks are problems to find the maximum or minimum of a function of a number of variables, which are subject to certain restrictions. Solving these problems is the issue in economics, public administration, military activities, industry etc. In other words, solving these problems means looking for an optimal allocation of limited resources with respect to the given reality.

2. DESCRIPTION OF THE PROBLEM

Thermal plant has the ability to work in three parallel branches. The working fluid can be used to operate one branch plant, as follows: when operating only the first branch, the

utilization of the system is such that it is possible to make a profit of 30 J/s (gain units) per unit volume of fluid, while 60% of the technological remainder can again be "recycled" under the same conditions. Working just the second branch brings 20 units of profit per unit volume of fluid and the reusability of 80% of raw materials. The work of the third branch brings a profit of 10 J/s per unit volume of fluid and the possibility of re-use 90% of raw materials. (The possibility of re-use means that it is such a technological process where at one time raw materials are only partially used, while the remainder of the raw materials can be used in the following period under the same conditions.)

We assume that the distribution takes 6 periods of time and that it starts with 100 units of volume of fluid. Based on the data obtained, one should determine the optimal distribution plan of the working fluid and total benefits, if at the end of the process the leftover material remains unutilised.

When designing the system, the most significant are the two cases:

- when at the start of a period, available raw material (working fluid) may be assigned to all three branches in a certain proportion, wherein one branch can be assigned a minimum of 10 units of volume of working fluid or a multiple of 10, because such are the technological requirements; and
- when at the start of a period, all of the fluid can be directed to a single branch.

3. MATHEMATICAL MODEL

The plant has three branches and therefore three possible directions of the working fluid. Let b_i denote the available quantity at the beginning of the i-th period, calculated from the end of the planning period. The amount of raw material (working fluid) assigned to individual branches at the beginning of the i-th period will be denoted by x_i, y_i, i z_i. These quantities must satisfy the obvious relation:

$$x_i+y_i+z_i=b_i$$

Depending on the amount of feedstock (incoming raw materials) in certain branches, profits are realized respectively by the branches during the i-th period:

Gain of the I branch: $f_1(x_i)$
Gain of the II branch: $f_2(y_i)$
Gain of the III branch: $f_3(z_i)$

Feedstock remainder is reassigned at the end of each period. This raw material reassignment is defined by the following branch functions:

Feedstock remainder of the I branch: $g_1(x_i)$
Feedstock remainder of the II branch: $g_2(y_i)$
Feedstock remainder of the III branch: $g_3(z_i)$

Thus, at the beginning of the i-th period there is available material for distribution:

$$b_{i-1}=g_1(x_i)+g_2(y_i)+g_3(z_i)$$

however, proceeding from the first equation, the expression for b_{i-1} can be written as:

$$b_{i-1}=g_1(x_i)+g_2(y_i)+g_3(b_i - x_i - y_i)$$

The assumption is that the profit is not invested in new production.

Proceeding from the above stated, a mathematical model of the problem can be formulated comprising the objective function

$$(\max)F(X,Y) = \sum_{i=1}^{N} [\, f_1(x_i) + f_2(y_i) + f_3(b_i - x_i - y_i)]$$

The task of optimizing the distribution process boils down to finding a vector:

$$X = (x_1 , x_2 , x_3 \dots\dots\dots x_N)$$
$$Y = (y_1 , y_2 , y_3 \dots\dots\dots y_N)$$

which provides the maximum of the function F(X,Y), at the same time complying with the limits defined by the expressions:

$$x_i + y_i \leq b_i, \quad (i = 1,2,\ldots\ldots N)$$
$$x_i \geq 0 \text{ i } y_i \geq 0,$$

where b_1 is a predetermined quantity.
The maximum gain depends only on the quantities:
b_N – initial available amount of raw materials at the beginning of the distribution that will last N periods,
N – number of periods in which distribution will be made.
Pursuant to the principle of optimality, one can form the corresponding recurrent relations through which the stated problem is solved.
With $W_1(b_1)$ denoting the optimal allocation gain (profit) for the case N=1, that is, the optimal allocation is sought for one period only, one can write:

$$W_1(b_1) = \max_{(x_1,y_1)} \{f_1(x_1) + f_2(y_1) + f_3(b_1 - x_1 - y_1)\}$$

where:

$$x_1 + y_1 \leq b_1\,, \quad x_1 \geq 0 \text{ i } y_1 \geq 0$$

If the planning period consists of two separate periods at the beginning of which a decision on the allocation of raw materials is made, then the optimal gain can be expressed as a function $W_2(b_2)$ as follows:

$$W_2(b_2) = \max_{(x_2,y_2)} \{f_1(x_2) + f_2(y_2) + f_3(b_2 - x_2 - y_2) + W_1(b_2)\}$$

where:

$$x_2 + y_2 \leq b_2\,, \quad x_2 \geq 0 \text{ i } y_2 \geq 0$$

Similar considerations lead to the general recurrent relation:

$$W_N(b_N) = \max_{(x_N,y_N)} \{f_1(x_N) + f_2(y_N) + f_3(b_N - x_N - y_N) + W_{N-1}(b_{N-1})\}$$

where:

$$x_N + y_N \leq b_N\,, \quad x_N \geq 0 \text{ i } y_N \geq 0$$

while N denotes the number of periods in the planning process of feedstock allocation to the three branches.

4. OPTIMIZATION OF THE PLANT'S OPERATION

The problem is solved using the dynamic programming approach, so that all calculations are based on the relations of multidimensional optimal distribution. The problem is solved successively, starting from the case of process taking only one time period. In that case, the optimal "policy" provides profits given by the expression:

$$W_1(b_1) = \max_{(x_1,y_1)} \{30x_1 + 20y_1 + 10(b_1 - x_1 - y_1)\} = \max_{(x_1,y_1)} \{20x_1 + 10y_1 + 10b_1\}$$

with constraints:

$$x_1 + y_1 \leq b_1, \quad x_1 \geq 0, \quad y_1 \geq 0$$

The calculated values for $W_1(b_1)$ are shown in Table 1.
If the process takes two time periods, the optimal "policy" provides profits given by the expression:

$$W_2(b_2) = \max_{(x_2,y_2)} \{20x_2 + 10y_2 + 10b_2 + W_1(b_1)\}$$

with constraints:

$$x_2 + y_2 \leq b_2, \quad x_2 \geq 0 \quad \text{and} \quad y_2 \geq 0$$

Quantity b_1 is calculated from the expression:

$$b_1 = 0{,}6x_2 + 0{,}8y_2 + 0{,}9(b_2 - x_2 - y_2) \quad \text{i.e.}$$
$$b_1 = 0{,}9b_2 - 0{,}3x_2 - 0{,}1y_2.$$

For each possible value of b_2, assessment of $W_2(b_2)$ boils down to finding a solution to the problem:

$$(\max)F_2 = 20x_2 + 10y_2 + 10b_2 + W_1(b_1)$$

provided that:

$$x_2 + y_2 \leq b_2$$
$$0{,}9b_2 - 0{,}3x_2 - 0{,}1y_2 = b_1$$
$$x_2 \geq 0 \quad \text{and} \quad y_2 \geq 0$$

The calculated values of $W_2(b_2)$ are given in Table 1.

Tabela 1.

b	$W_1(b_1)$	x_1	y_1	$W_2(b_2)$	x_2	y_2	$W_3(b_3)$	x_3	y_3	$W_4(b_4)$	x_4	y_4	$W_5(b_5)$	x_5	y_5	$W_6(b_6)$	x_6	y_6
0	0	0	0															
10	300	10	0															
20	600	20	0	900	20	0												
30	900	30	0	1400	20	10	1700	20	10									
40	1200	40	0	1800	10	30	2300	10	30	2600	10	30						
					40	0												
50	1500	50	0	2400	50	0	2900	50	0	3300	0	50	3600	0	50			
60	1800	60	0	2800	40	20	3400	40	20	3900	40	20	4200	40	20	4200	40	20
70	2100	70	0	3300	70	0	4100	30	40	4600	30	40	5000	30	40	5300	30	40
80	2400	80	0	3800	70	10	4700	70	10	5200	20	60	5700	20	60	6000	20	60
											70	10						
90	2700	90	0	4200	60	30	5200	10	80	6000	10	80	6500	10	80	6900	10	80
					90	0		60	30									
100	3000	100	0	4800	100	0	5800	50	50	6700	0	100	7200	0	100	7700	0	100
								100	0									
								0	100									

For example, this is a simplification that can be used to solve optimization problems in the process of calculating $W_2(b_2)$. For b_2=40 the results are:

$$\begin{aligned} x_2 &= 0, \quad 10, \quad 20, \quad 30, \quad 40, \\ y_2 &= 40, \quad 30, \quad 20, \quad 10, \quad 0, \\ b_1 &= 32, \quad 30, \quad 28, \quad 26, \quad 24, \\ F_2 &= 1700, \quad 1800, \quad 1600, \quad 1700, \quad 1800. \end{aligned}$$

So $(\max)F_2 = 1800$ for $x_2 = 40$, $y_2 = 0$, and the second solution is $x_2 = 10$, $y_2 = 30$.
When the process takes two time periods, the optimal "policy" provides profits given by the expression:

$$W_3(b_3) = \max_{(x_3,y_3)} \{20x_3 + 10y_3 + 10b_3 + W_2(b_2)\}$$

provided:

$$x_3 + y_3 \leq b_3, \quad x_3 \geq 0 \quad \text{and} \quad y_3 \geq 0$$

Magnitude b_2 is calculated from:

$$b_2 = 0{,}6x_3 + 0{,}8y_3 + 0{,}9(b_3 - x_3 - y_3)$$

respectively:

$$b_2 = 0{,}9b_3 - 0{,}3x_3 - 0{,}1y_3$$

In a similar way as in the calculation of the $W_2(b_2)$, for each possible value of b_3 a dynamic programming problem is defined :

$$(\max)F_3 = 20x_3 + 10y_3 + 10b_3 + W_2(b_2)$$

provided that:

$$\begin{gathered} x_3 + y_3 \leq b_3 \\ 0{,}9b_3 - 0{,}3x_3 - 0{,}1y_3 = b_2 \\ x_3 \geq 0 \quad \text{and} \quad y_3 \geq 0 \end{gathered}$$

In a similar manner $W_4(b_4)$, $W_5(b_5)$ and $W_6(b_5)$ were calculated. The results are recorded in Table 1.
The analysis of the results obtained leads to the conclusion that the maximum profit from the utilization of 100 units of raw materials equals:

$$W_6(b_6) = W_6(100) = 7700 \text{ gain units}$$

Table 2 shows the allocation plan and profits per period; it was drawn up on the basis of the results in Table 1.

Table 2

i	D_i	b_i	x_i	y_i	$z_i = b_i - x_i - y_i$
6	2000	100	0	100	0
5	1800	80	20	60	0
4	1600	60	40	20	0
3	900	40	10	30	0
2	800	30	20	10	0
1	600	20	20	0	0
TOTAL	7700				

Column D_i shows the allocation plan and profits per period, while columns x_i, y_i and z_i show optimal feedstock allocation plan of raw materials to individual processes during the sixth period. This allocation plan provides an overall gain of 7700 units.

5. THE SECOND PLANT OPERATING MODE

If the problem is set up so that at the start, available raw materials can be allocated to one process only, where the allocation is done under the same conditions as in the previous case, namely, that the feedstock can be allocated throughout the periods in multiples of 10 units, profits will be lower. And the problem can be solved in a similar manner.

The starting point is the case that there is only one period until the end of the utilization of raw materials and that at the beginning of this period one disposes of b_1 units, optimal profits will be defined by:

$$W_1(b_1) = \max\begin{Bmatrix} 30b_1 + 0 \cdot 0{,}6b_1 \\ 20b_1 + 0 \cdot 0{,}8b_1 \\ 10b_1 + 0 \cdot 0{,}9b_1 \end{Bmatrix} = 30b_1$$

Table 3 shows the changes in the gain depending on the value of b_1.

Table 3.

b	$W_1(b_1)$	Process	$W_2(b_2)$	process	$W_3(b_3)$	process	$W_4(b_4)$	process	$W_5(b_5)$	process	$W_6(b_6)$	process
0	0		0		0		0		0		0	
10	300	1	300	1	300	1	300	1	300	1	300	1
20	600	1	900	1	900	1	900	1	900	1	900	1
30	900	1	1200	1 2	1500	2	1500	2	1500	2	1500	2
40	1200	1	1800	1	2100	1	2300	2	2300	2	2300	2
50	1500	1	2400	1	2800	2	3100	2	3300	2	3300	2
60	1800	1	2700	1	3000	1 2 3	3400	3	3700	3	3900	3
70	2100	1	3300	1	3900	1	4200	1 2	4500	2	4700	2
80	2400	1	3600	1	4300	2	4700	3	5000	2 3	5300	2 3
90	2700	1	4200	1	5100	1 2	5700	2	6000	2	6300	2
100	3000	1	4800	1	5700	1	6300	2	6700	2 3	7000	2 3

In table 3, digits “1“, “2“ and “3“ indicate the possible thermal plant operation modes:

"1" – mode that brings 30 units of profit per unit volume of fluid and the recovery of 60% of the raw materials.
"2" – mode that brings 20 units of profit per unit volume of fluid and the recovery of 80% of the raw materials.
"1" – mode that brings 10 units of profit per unit volume of fluid and the recovery of 90% of the raw materials.

Table 4.

Periods	**Plan 1.**		**Plan 2.**		**Plan 3.**		**Plan 4.**		**Plan 5.**	
	D_i	Applied process	D_i	Applied process	D_i	Applied process	D_i	Applied process	D_i	Applied process
6	2000	2	1000	3	2000	2	1000	3	2000	2
5	1600	2	1800	2	800	3	1800	2	800	3
4	600	3	2100	1	2100	1	1400	2	1400	2
3	1000	2	1200	1	1200	1	1000	2	1000	2
2	1200	1	600	1	600	1	1200	1	1200	1
1	600	1	300	1	300	1	600	1	600	1
TOTAL	7000		7000		7000		7000		7000	

When considering a process that takes two periods, the optimal profits will be defined by the expression:

$$W_2(b_2) = \max\begin{Bmatrix} 30b_2 + W_1(0{,}6b_2) \\ 20b_2 + W_1(0{,}8b_2) \\ 10b_2 + W_1(0{,}9b_2) \end{Bmatrix}$$

For b_2=(0, 10, 20, ……,100), the $W_2(b_2)$ values are calculated

$$W_2(0) = 0$$

$$W_2(10) = \max\begin{Bmatrix} 30 \cdot 10 + W_1(6) \\ 20 \cdot 10 + W_1(8) \\ 10 \cdot 10 + W_1(9) \end{Bmatrix} = 300$$

$$W_1(b) = 0 \quad za \quad b < 10$$

$$W_2(20) = \max\begin{Bmatrix} 30 \cdot 20 + W_1(12) \\ 20 \cdot 20 + W_1(16) \\ 10 \cdot 20 + W_1(18) \end{Bmatrix} = 900$$

$$\text{because } W_1(b) = 30 \quad \text{for} \quad 10 > b < 20$$

The other values of $W_2(b_2)$ were similarly calculated and recorded in Table 3.

For the process that takes three, four, five or six periods, analog expressions are formed, calculation procedure being identical. The results of this calculation are given in Table 3. In this case, as can be seen from the results in Table 3, the maximum profit from the utilization of 100 units of raw materials:

$$W_6(100) = 7000 \text{ gain units}$$

The optimal plan of raw materials allocation is not unique. Solution variants are shown in Table 4, together with the amount of profit that can be accomplished in certain periods.

Among all the plans of using the working fluid which will provide the same gain, plan 4 could be selected as the most suitable one in terms of uniformity of profits at certain periods.

6. CONCLUSION

Optimal use of energy facilities is extremely important because of the energy crisis, the consequences of which are already very visible, as well as due to financial factors, which the owner finds singularly important. Technical — economic indicators justify the analysis and calculation of parameters of the system. Optimal use of the possibilities of power plants also enables the maximizing of the other functions of the whole system.

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