The occurrence of ice accretion, formed during the winter months on overhead, hanging power lines of the electric power network, can lead to disasters, collapse of load-bearing structures, transmission lines and poles, due to the increased cable weight, especially in combination with the effect of wind and snow. These processes are more often appeared in Serbia and the goal of this master thesis is to consider and analyze the physics of the icing process. It also contains mathematical models which describe process of formation and growth ice load on horizontal cylindrical surfaces and method how these problems can be exceeded.

1 Introduction

There are two cases of freezing rain droplets in contact with solid surfaces. The first case is the case of so-called freezing rain (Figure 1), when the droplets of supercooled water (temperature below freezing point) contacts with solid surface. Due to unstable energy state of these water droplets, despite the temperature of solid surface that can be above freezing point, liquid phase of water transforms into solid phase. Another case of freezing rain droplets which contacts with solid surfaces occurs when the temperature of these droplets is above freezing point and the temperature of solid surface is below freezing point. Regardless of the way of forming ice accretion on power lines, this can cause great damages (Figure 2). Many numerical, analytical and empirical models were developed, but none of them are both correct and complete in predicting design glaze ice load (Imai, 1953; Lenhard, 1955; Chainé and Castonguay, 1974; Anon, 1984; Goodwin et al., 1983; Lozowski et al., 1983; Makkonen, 1984).

Figure 1: Freezing rain diagram

Figure 2: Damage caused by ice storm
2 Description of the model

In order to determine the analytical model to predict the location of initial freezing we consider the case when the droplet temperature is above the freezing point. Power line is presented as horizontal cylinder which is exposed to vertically falling rain droplets (Figure 3). The cylinder has zero heat capacity and thermal conductivity and the air temperature is below freezing.

\[ T_{\text{atm}} \]  
\[ T_k \]
\[ T_{p0} \]
\[ T_p \]
\[ T_{\text{led}} \]
\[ \Delta q_m \]
\[ q_m \]
\[ \vartheta \]
\[ R \]

Figure 3: A horizontal cylinder exposed to vertically falling rain droplet

- \( T_{\text{atm}} \) (°C) is the air temperature, \( T_k \) (°C) is the droplet temperature, \( T_p \) (°C) is the local surface temperature, \( \vartheta \) (rad) is the angle measured from the top of the cylinder, \( T_{p0} \) (°C) is the surface temperature for \( \vartheta = 0 \), \( T_{\text{led}} \) (°C) is the freezing temperature, \( R \) (m) is cylinder radius, \( q_m \) (kg/ms) is mass flux of water flowing along the cylinder and \( \Delta q_m \) (kg/m²s) is rainfall rate.

If the air or drop temperature is sufficiently high, the freezing process may not begin at the upper half of the cylinder. The water film flows down the cylinder and cools because of convective heat loss to the air, and at some location ice may begin to form. The mass and heat balance equations for the upper and lower halves of the cylinder are considered separately because the impinging droplets have the influence on the upper half of the cylinder.

3 Results

It is assumed that droplets hit the cylinder without shedding. The mass balance equation for the upper half of the cylinder is determined as:

\[
\frac{ds}{d\theta} = \Delta q_m \cos \theta
\]

where \( s \) (m) is distance along the surface (\( ds = Rd\theta \)),

By the integration of equation (1) it is obtained:

\[
q_m = \Delta q_m R \sin \theta
\]

where \( 0 \leq \theta \leq \frac{\pi}{2} \)

It is assumed that there is no radial temperature gradient inside the water film and if kinetic energy of air and water, heat released by evaporation and heat released by the runback water are neglected, heat balance equation can be written by the equation (3):

\[
c_w q_m dT_p = c_w \Delta q_m (T_k - T_p) R \cos \theta d\theta + h(T_{\text{atm}} - T_p) Rd\theta
\]

where \( c_w \) (J/kgK) is specific heat capacity of water, \( h \) (W/m²K) is the heat-transfer coefficient which includes the combined effects of convection, evaporation and radiation.

Equation (3) states that the rate of change of water's internal energy depends on the sensible heat of warm droplets which vertically fall on the surface of the cylinder and the convective heat loss to the air of the temperature below the freezing.

In order to non-dimensionalize the model, the heat balance equation is written for the upper half of the cylinder as:

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where $T_{po}$ (°C) is the surface temperature for $\theta=0$.

Using equations (2) and (4) the dimensionless form of the equation (3) is obtained:

$$\Theta' + \frac{A \cos \theta}{\sin \theta} - (A + 1) \frac{\cos \theta}{\sin \theta} = 0$$

(5)

where $A = \frac{h}{c_w \Delta q_m}$ is constant and $\Theta = \frac{T_{po} - T_{atm}}{T_{po} - T_{atm}}$ is the dimensionless surface temperature.

Equation (5) is first ordered linear differential equation which can be solved by Fourier’s method of separation and has an analytical solution for $A=1$:

$$\Theta(\Theta) = u(\theta) \cdot v(\theta) = \frac{\ln(1+\cos \theta) - \cos \theta + 1 - \ln 2}{0.5(1-\cos \theta)}$$

(6)

Solution of the equation (5) represents the distribution of film temperature over the upper half of the cylinder.

If freezing doesn’t begin at $\theta=\pi/2$, it is necessary to consider the heat balance equation on the lower half of the cylinder. It can be shown by equation (7):

$$c_w \Delta q_m R \frac{dT_p}{d\theta} = h(T_{atm} - T_p) R d\theta$$

(7)

where $\frac{\pi}{2} \leq \theta \leq \pi$

The distribution of the dimensionless surface temperature on the cylinder for the range $\pi/2 \leq \theta \leq \pi$ is obtained by the integration of the equation (7):

$$\Theta(\theta) = e^{-A(\theta - \frac{\pi}{2})}$$

(8)

Figure 4: Dimensionless surface film temperature around the cylinder (equations (6) and (8))

An increase of the parameter $A$ leads to rapid decrease of the dimensionless surface temperature of the film water with distance along the surface of the cylinder. High values of the parameter $A$ correspond to rapid heat exchange with cold air or to a small mass flux of impinging rain droplets. It should be emphasized that this distribution of the temperature is valid only for the area where the surface temperature is above freezing.

It is necessary to consider boundary case when sensible heat released by runback water is neglected to determine the location on the cylinder where freezing begins ($\theta_C$). Heat loss to the air is in the balance with sensible heat secured by impinging warm droplets:
\[ h(T_{\text{led}} - T_{\text{amb}}) = c_w \Delta q_m(T_k - T_{\text{led}}) \cos \theta_C \]  

(9)

In this case, the location where freezing begins is given by the critical freezing angle:

\[ \theta_C = \arccos \alpha \]  

(10)

where \( \alpha = \frac{h(T_{\text{led}} - T_{\text{amb}})}{c_w \Delta q_m R_k (T_k - T_{\text{led}})} \) is heat-ratio parameter.

If the heat-ratio parameter is greater than unity, the freezing begins at \( \theta = 0 \). On the other hand, when the heat-ratio parameter has a small value, location of the initial freezing approaches \( \pi/2 \).

The mass of ice \( m_{\text{led}} \) (kg/m) which forms during a time interval \( t \) (s) is proportional to the difference between the convective heat flux from the cylinder and the sensible heat flux of the warm droplets:

\[ m_{\text{led}} = 2 \pi R h \frac{R_{\text{led}}}{c_w \Delta q_m} (T_k - T_{\text{led}}) \]  

(11)

where \( r_{\text{led}} \) (J/kg) is heat gained for phase changes and \( T_{\text{led}} \) (°C) is the freezing temperature.

The volume of ice per length meter is equal to the surface of the ice accretion which is perpendicular to the cylinder axis, i.e. surface of the cross section of cylinder area:

\[ \frac{m_{\text{led}}}{\rho_1} = (R_1^2 - R^2) \pi \]  

(12)

where \( \rho_1 \) (kg/m³) is density of ice, \( R_1 \) (m) is the radius of the iced cylinder and \( R \) (m) is the cylinder radius.

Equation (12) gives the thickness of the ice layer \( \Delta R = R_1 - R \) (m):

\[ \Delta R = (R_1^2 + \frac{\rho_{\text{led}}}{\rho_1})^{0.5} - R \]  

(13)

The mass of ice is very important quantity because power cables and towers may be damaged or destroyed due to the added burden of the ice. The United States, Canada, France, Russia, Japan, Korea, etc. are frequently attacked by the icing in the world. Serbia was also attacked by icing in 2014.

There are traditional and modern ways to deal with ice. The traditional way are based on anti-icing and modern way prevents ice formation. This problem can be solved by heating to a temperature of wires + (10-20°C) until the ice. The way of heating air electricity transmission line can be by spreading it on a running high-frequency electromagnetic waves (skin effect).

4 Conclusion

There are two cases of freezing rain droplets in contact with solid surfaces. The first case is the case of so-called freezing rain, when the droplets of supercooled water (temperature below freezing point) contacts with solid surface. Another case of freezing rain droplets occurs when the temperature of these droplets is above freezing point and the temperature of solid surface is below freezing point.

In this master thesis the analytical model is developed which calculates the angular distribution of the water-film temperature, location on the cylinder where freezing begins, the mass as well as the thickness of the ice.

The icing of overhead electrical transmission lines may cause a many problems such as overloading, non-uniform icing, wire galloping, damaged power cables and towers. This is the reason why this occurrence should be a new objective of the environmental thermophysics study.
