PRORAČUN ČELIČNIH SANDUČASTIH MOSTOVA SA ZAKRIVLJENOM DONJOM FLANŠOM

STEEL BOX-GIRDER BRIDGE DECKS WITH CURVED BOTTOM FLANGE – DESIGN RULES

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Ovaj rad se bavi proračunom nosivosti na savijanje, smicanje, kao i interakciju momenta i smicanja, u slučaju sandučastih čeličnih mostova sa zakrivljenom donjom flanšom. Dva analitička modela, za proračun nosivosti na savijanje i smicanje, su predložena po uzoru na modele proračuna koji figurišu u tekućem standardu za čelične mostove (Eurokod 3), s tim što umesto trapezoidnih sandučastih mostova sa ravnom donjom flanšom, predloženi modeli uzimaju u obzir prisustvo ukrućenih cilindričnih ljuski u donjem delu preseka. Predloženi modeli su potom inplementirani u formulu za proračun interakcije moment-smičuća sila (M-V), čija je validnost proverena koristeći prethodno dobijene numeričke rezultate (FEM). Rezultati ukazuju da se M-V formula može sa velikom sigurnošću koristiti za sve geometrijske opsege razmatrane u ovom radu. Konačno, predložene formule su procenjene statistički, proračunom parcijalnog koeficijenta sigurnosti (γ_M). Ustanvoljeno je da se parcijalni faktori za trapezoidne poprečne preseke ($\gamma_{M,0} = 1.0$ i $\gamma_{M,1} = 1.1$) mogu koristiti i u formulama za procenu nosivosti čeličnih mostova sa zakrivljenom donjom flanšom.

Ključne reči: *zakrivljene čelične ploče, sandučasti gredni mostovi, M-V interakcija, proračunska nosivosti*

In this paper, design rules are proposed for the assessment of the bending, shear and M-V interaction capacity of box-girder bridge decks with a curved bottom flange. The two analytical design models, for the determination of the bending and shear resistance, use the EC3 format for straight bottom flanges adapted to account for the presence of a stiffened curved panel in the bottom flange. Subsequently, these two models are implemented in the M-V interaction equation, adopted in prEN 1993-1-5, whose safety is verified against the FEM results. The results show that the M-V interaction equation might be safely applied across the entire range of parameters covered in this study. Finally, the proposed design rules are statistically assessed by calculating the partial factor (γ_M). It is concluded that the partial factors $\gamma_{M,0} = 1.0$ and $\gamma_{M,1} = 1.1$ for the bending and shear resistance models, respectively, may be kept also in the design of box-girder bridges with a curved bottom flange.

Keywords: *curved steel panels; box-girder bridges; M-V interaction; design rules*

1. INTRODUCTION

The use of curved steel panels in bridge deck applications has recently become a very popular solution, both for aesthetic and structural reasons [1]. As many as 22 bridge decks have been built worldwide, mainly in urban environments where aesthetics plays an important role, as shown in Fig. 1.

The major difference between these bridge decks and conventional trapezoidal box-girder decks is the presence of a longitudinally stiffened curved panel in the bottom flange, as indicated in Fig. 2. Based on the figure, t and R are the thickness and the radius of the panel, a and b are the length and width of the panel, β is the sectorial angle defined as the ratio between the width and the radius of the panel. In addition, in case of a stiffened panel, a_{loc} is the length of the subpanel that corresponds to the distance between transverse stiffeners; b_{loc} is the width of the subpanel that corresponds to the

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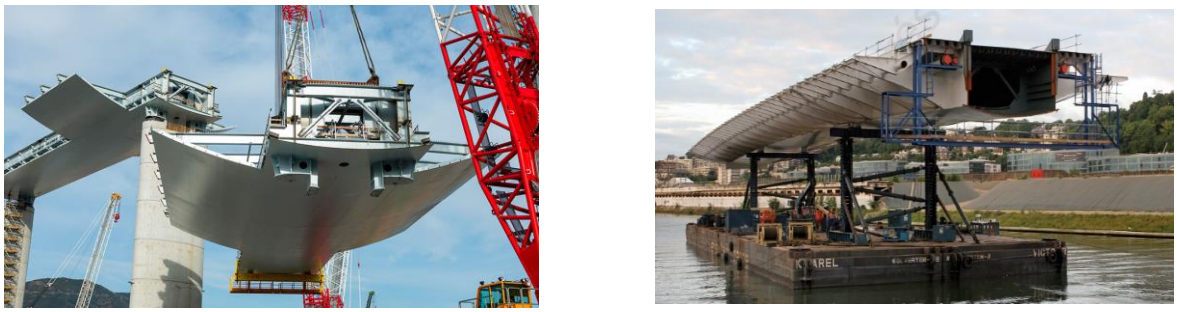
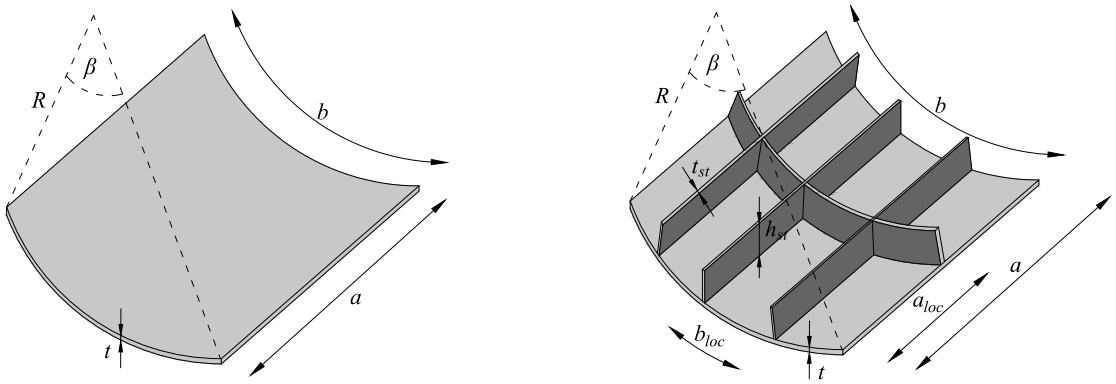
distance between longitudinal stiffeners, whereas t_{st} and h_{st} are the thickness and the height of the longitudinal stiffeners.



a) *b)*

Figure 1 – Examples of bridge decks with curved bottom flange: a) New Genoa bridge [2]; b) Renault bridge [3]



a) *b)*

Figure 2 – Geometry and notation of cylindrically curved panels: a) unstiffened; b) stiffened

In the literature, the geometry of a curved panel is commonly defined by two non-dimensional parameters: aspect ratio ($\alpha = a/b$) and curvature parameter ($Z = b^2/Rt$). Owing to the presence of longitudinal stiffeners in most cases of steel bridge decks, besides the global aspect ratio (α) and the curvature parameter (Z), it is necessary to define the local aspect ratio (α_{loc}) and curvature parameter (Z_{loc}), using the width and length of the corresponding sub-panel (between longitudinal stiffeners). The ranges of the most relevant geometrical parameters in the surveyed 22 bridges are summarized in Table 1.

Table 1 – Ranges of geometrical parameters of curved panels used in bridges

	Z	α	b/t	Z_{loc}	α_{loc}	b_{loc}/t
Min	16	0.4	75	0.2	2.67	10
Max	580	1.35	250	13.4	10	75
Average	100	0.7	165	3	4	28

One of the main reasons for the limited use of curved steel panels in bridge deck applications is the lack of design rules for the assessment of the buckling strength of curved panels since the scope of the actual design code for plated elements EN 1993-1-5 [4] is restricted to flat plates elements or considered nearly flat only (*i.e.* $Z \leq 1$). Hence, numerous studies on curved panels as an individual (isolated) structural element have been recently reported (summarized in [5]) and the design rules were proposed for the assessment of the curved panel resistance under several fundamental in-plane loads (*e.g.* axial compression, shear load and the combination of these two). Hence, this paper aims to extend the existing design rules by proposing new rules that use the same mathematical formalism from the actual code EN 1993-1-5 [4], modified to account for the presence of curved panels in the bottom flange. More specifically, instead of proposing a new M-V interaction equation, the objective

is to keep the one proposed and *adopted in prEN 1993-1-5* [6], given by Eq.(1) and to assess its applicability in the case of bridge decks with a curved bottom flange.

$$\eta_1 + \left[1 - \frac{M_{f,Rd}}{M_{eff,Rd}}\right]\left(2\bar{\eta}_3 - 1\right)^{\beta} \le 1.0 \qquad \text{with} \qquad \beta = \left(\frac{M_{f,Rd}}{M_{eff,Rd}} + 0.2\right)^{15} + 1 \tag{1}$$

where η_1 is the ratio between the applied (M) and the effective resistant bending moment ($M_{,eff,Rd}$); $\bar{\eta}_3$ is the ratio between the applied (V) and the shear buckling resistance of the web panel alone ($V_{bw,Rd}$), and $M_{f,Rd}$ is the bending resistance of the cross-section considering the effective area of the flanges alone.

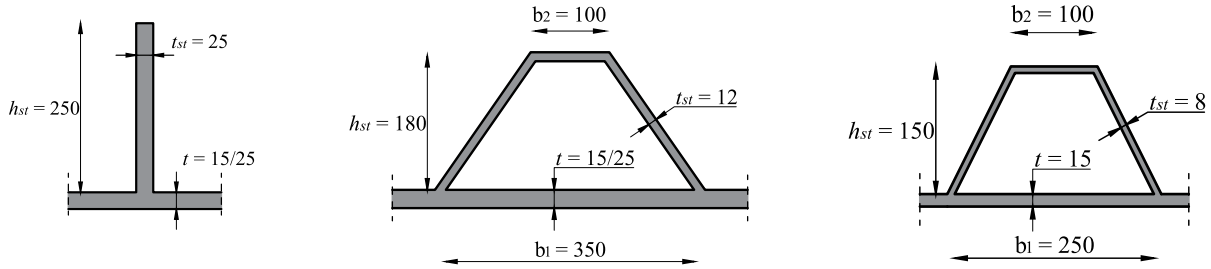
However, for the application of the M-V interaction equation, it is necessary first to define the effective bending resistance ($M_{eff,Rd}$) and the web shear resistance ($V_{bw,Rd}$), which is found to be the main challenge and the main motivation for the present work since there are still no analytical design rules applicable for this cross-section typology.

2. NUMERICAL PARAMETRIC STUDY

For the sake of the assessment of the proposed design rules in this paper, a numerical study is carried out, in which 210 different bridge cross-section configurations were considered, with the following ranges of parameters commonly encountered in bridge application:

- Z = 0, 50, 100, 200, 300 (42 each)
- α = 0.5, 1.0, 1.5 (70 each)

Regarding the bottom curved flange, besides the flat stiffeners (h_{st}/t_{st} = 250/25 mm), two longitudinal stiffener typologies were considered – trapezoidal stiffeners type 1 and type 2, as presented in Fig. 3.



a) *b)* *c)*

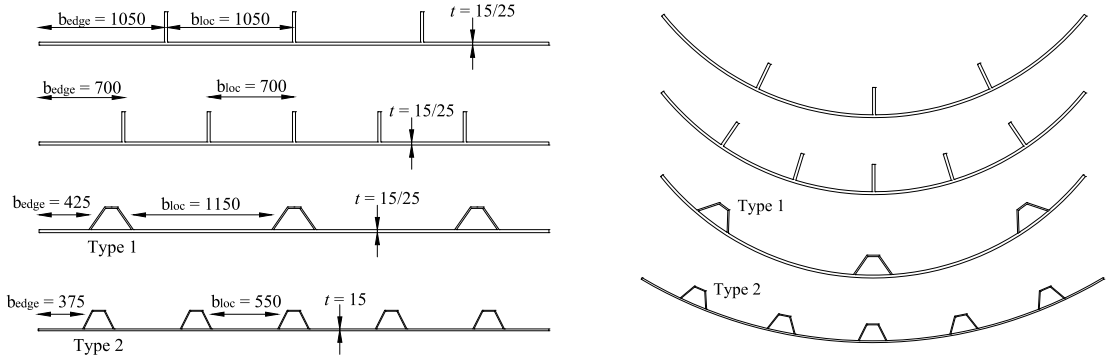
Figure 3 – Stiffeners: a) flat; b) trapezoidal– Type 1 and c) trapezoidal – Type 2

For each of the presented typologies, two configurations of longitudinal stiffeners were considered, with n_{st} = 3 and n_{st} = 5, where the distances between the stiffeners (b_{loc} and b_{edge}) were kept constant, as shown in Fig. 4. The b/t-ratio of the curved panels was varied in a wide range (up to 70), whereas the dimensions of the stiffeners are chosen to be class 3 (or lower), thus isolating merely the local buckling of the curved sub-panels.

As for the flat webs, two thickness were considered (t_w = 15 mm and t_w = 25 mm). The thickness of 15 mm was found sufficiently thin for the occurrence of the shear buckling of the webs. Regarding the longitudinal stiffeners in the webs, the same typologies used for the bottom flange were adopted, whereas the stiffeners were placed at 1/3 and 2/3 of the web height. All the other relevant information used for the definition of the FE model (*e.g.* static scheme, material, imperfections, mesh size, etc.) were discussed in [1].

For each of 210 box-girder configurations, ten different M/V-ratios were applied in the studied cross-section, comprising a total of 2100 numerical simulations. Two extreme M/V-ratios correspond to pure bending and shear resistances, while the other eight represent intermediate points of the interaction. The main output of the parametric study are numerically generated M-V interaction diagrams for various box-girder bridge cross-sections, serving as the benchmark for the safety verification of

design rules proposed in this paper. In Fig. 5, the generated M-V interaction diagrams are presented for two examples. For more details on the results, the reader is suggested to read Ljubinkovic [1].



a) *b)*

Figure 4 – Stiffener configurations: a) Z = 0; b) Z = 300

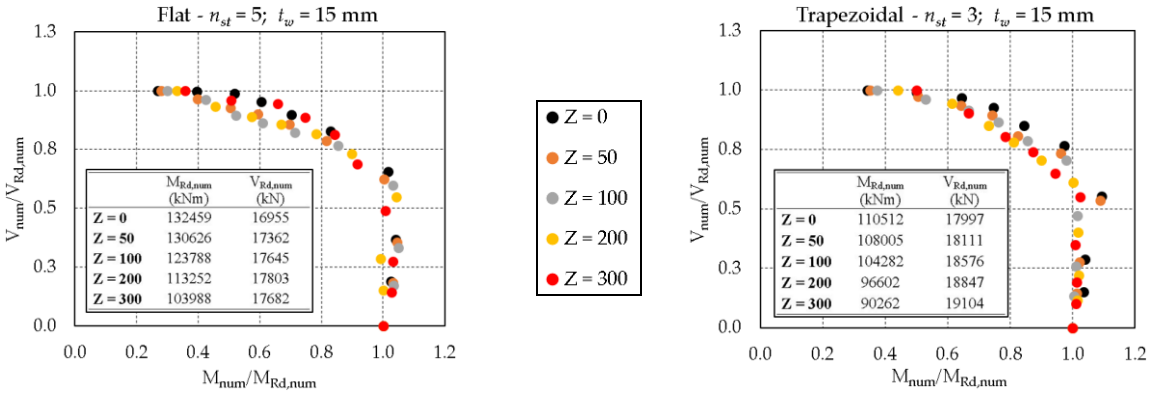


Figure 5 – M-V interaction diagrams for different cross-section configurations

3. BENDING RESISTANCE MODEL

According to EN 1993-1-5, the bending resistance of a class 4 box-girder cross-section ($M_{eff,Rd}$) is given by Eq.(2), where f_y is the yield strength, γ_{M0} is the partial factor for cross-section checks defined by the standard as γ_{M0} = 1.0, and W_{eff} is the effective section modulus. For the determination of the effective properties of the stiffened bottom flange (and web), the standard accounts for the local and global buckling behaviour, while in the latter both plate-like and column-like behaviour is considered (see §4.5 of EN 1993-1-5).

$$M_{eff,Rd} = \frac{W_{eff} f_y}{\gamma_{M0}} \tag{2}$$

For the calculation of the bending resistance of a class 4 box-girder cross-section ($M_{eff,Rd}$) with a curved bottom flange, a simplified design model is proposed in this paper, called *Curved panel approach*. It represents an extension of the corresponding design model from EN 1993-1-5 for trapezoidal box-girder bridge cross-sections, where the calculation of the effective modulus W_{eff} is presented in a flowchart in Fig. 6.

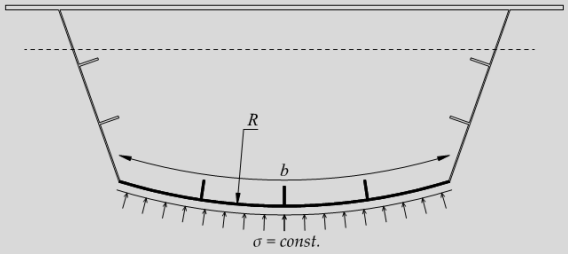
It is noted that in Step 1, using the analogy with the method from EN 1993-1-5, a simplification is adopted, according to which the entire curved panel is assumed to be the bottom flange, subjected to uniform axial compression (ψ = 1.0). Such simplification, although conservative, is very suitable for widespread engineering practice since it corresponds to what is currently done in EN 1993-1-5. Moreover, in Step 2, the method proposed within the OUTBURST research project [7], for the estimation of the compressive resistance of the stiffened curved is adopted, being the most suitable as revealed in a comparative study reported in [8].

Step 1

Definition of the curved flange and the web

Bottom flange: entire curved panel (Assumed to be under uniform axial compression - $\psi = 1.0$).

Web: part between the top and the bottom flange



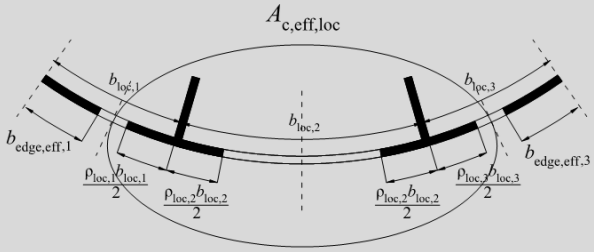
Step 2

Effective area of the stiffened bottom flange - OUTBURST method [7]

I. Local buckling: (ρ_{loc})

II. Global buckling:
- shell-type (ρ)
- column-type (χ_c)
- interaction(shell+column):(ρ_c)

III. Effective area: (A_{eff})



Step 3

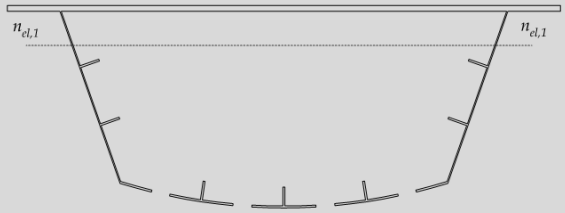
Definition of a new position of the neutral axis

Cross-section consists of:

(1) the gross area of the top flange,

(2) the gross area of the webs, and

(3) the effective area of the bottom flange.



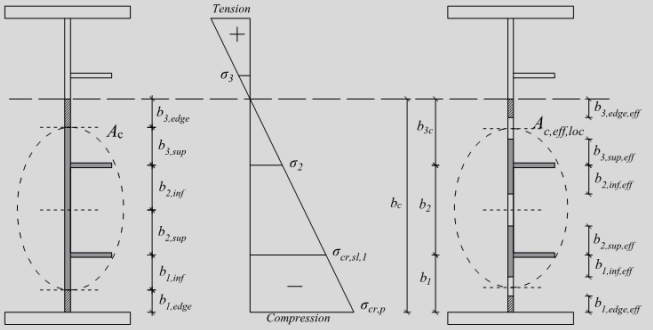
Step 4

Effective area of the stiffened web - Annex A of EN 1993-1-5 [4]

I. Local buckling: (ρ_{loc})

II. Global buckling:
- plate-type (ρ)
- column-type (χ_c)
- interaction(plate+column):(ρ_c)

III. Effective area: (A_{eff})



Step 5

Definition of W_{eff} for a new position of the neutral axis

Cross-section consists of:

(1) the gross area of the top flange,

(2) the effective area of the bottom flange, and

(3) the effective area of the web.

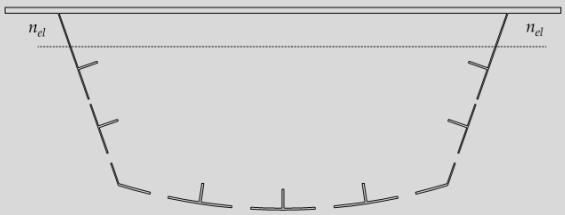


Figure 6 –Flowchart – calculation of the effective section modulus W_{eff}

4. SHEAR RESISTANCE MODEL

Based on the results from the numerical study, it is noticed that the curved bottom flange also contributes to the development of shear stresses, which may be seen also in Fig. 7, where the distribution of the shear stresses at the moment of failure are plotted for various curvature parameters.

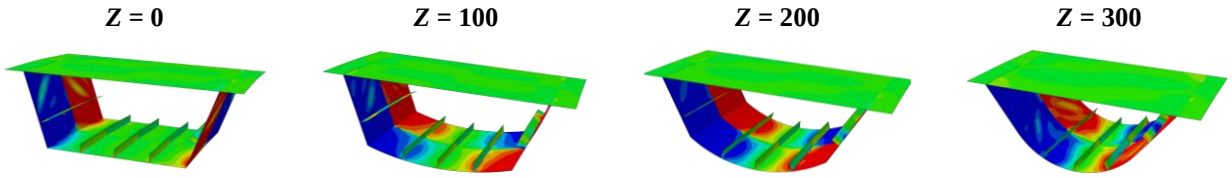


Figure 7 –Shear stress distribution in cross-sections

This leads to the conclusion that the total shear resistance of a cross-section, besides the flat part of the web with depth $h_{w,flat}$, should be increased for an extra portion of the bottom curved flange, with the length marked as L_w in the remaining of the document. Hence, a new design resistance model is hereby proposed, called *Extended web model*, according to which, the height of the flat web, for which the resistance is calculated, is elongated from $h_{w,flat}$ to h_w, as shown in Fig. 8.

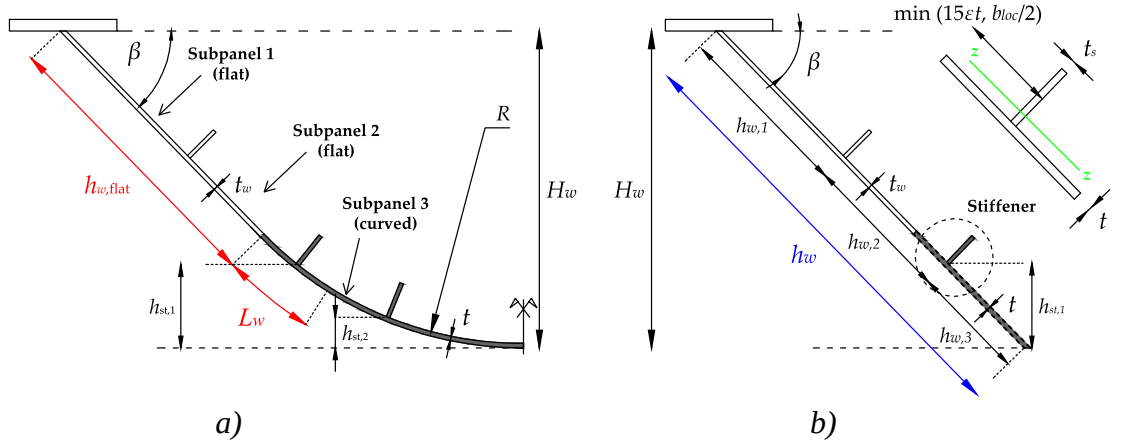


Figure 8 –Proposed shear resistance model: a) real web geometry; b) extended flat web

Namely, h_w is obtained as the projection of the cross-section total depth H_w onto the plane of the flat part of the web (*i.e.* $h_w = H_w/\sin\beta$). Using the notations from Fig. 8, the characteristic shear buckling resistance of the web panel alone (without flange contributions) may be calculated by Eq.(3) as:

$$V_{bw,Rk} = \frac{\chi_w \left[h_{w,flat} t_w + (h_w - h_{w,flat}) t \right] f_y}{\sqrt{3}} \le V_{pl,Rk} \tag{3}$$

where $V_{pl,Rk}$ is the characteristic plastic shear resistance, given by Eq.(4),

$$V_{pl,Rk} = \frac{\left(h_{w,flat} \cdot t_w + L_w \cdot t \right) f_y}{\sqrt{3}} \tag{4}$$

where $h_{w,flat}$ is the length of the flat part of the web, t and t_w are thicknesses of the bottom flange and flat web, respectively, and L_w is the part of the bottom flange that participates in the shear plastic resistance, calculated by Eq.(5) as:

$$L_w = \frac{1}{R}\left(\frac{b}{\pi}\right)^2 = \frac{Z \cdot t}{\pi^2} \tag{5}$$

whereas χ_w is the shear reduction factor obtained from Eqs.(6)-(7) of EN 1993-1-5, with the slenderness $\bar{\lambda}_w$ calculated using the flowchart in Fig. 9.

$$\chi_w = \begin{cases} \eta & \text{if} \quad \bar{\lambda}_w < \dfrac{0.83}{\eta} \\ \dfrac{0.83}{\bar{\lambda}_w} & \text{if} \quad \dfrac{0.83}{\eta} \le \bar{\lambda}_w < 1.08 \\ \dfrac{1.37}{0.7+\bar{\lambda}_w} & \text{if} \quad \bar{\lambda}_w \ge 1.08 \end{cases} \tag{6}$$

$$\bar{\lambda}_w = \max\left(\frac{h_w / t_w}{37.4 \cdot \varepsilon \cdot \sqrt{k_\tau}}; \frac{b_{loc,i} / t_i}{37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau,i}}} \right) \tag{7}$$

where k_τ is the shear buckling coefficient, calculated in accordance with Annex A.3 of EN 1993-1-5. In the current study, only webs with 1 or 2 longitudinal stiffeners and an aspect ratio of less than $\alpha < 3.0$ are studied, hence, the expressions for unstiffened and stiffened plates are given by Eq.(8) and Eq.(9), respectively

$$k_\tau = \begin{cases} 4.0 + 5.34\left(\dfrac{1}{\alpha}\right)^2 & \text{if} \quad \alpha < 1.0 \\ 5.34 + 4.0\left(\dfrac{1}{\alpha}\right)^2 & \text{if} \quad \alpha \ge 1.0 \end{cases} \tag{8}$$

$$k_\tau = 4.1 + \frac{6.3 + 0.18\dfrac{I_{sl}}{t_w{}^3 h_w}}{\alpha^2} + 2.2\sqrt[3]{\frac{I_{sl}}{t_w{}^3 h_w}} \tag{9}$$

In this flowchart, a generic example from Fig. 8 is considered, in which the extended web has two longitudinal stiffeners (one on the flat part of the web and the other on the curved within the length L_w), thus the extended web consists of three subpanels. This means that besides the global critical stress and slenderness of the entire 2-stiffened web, for each of three subpanels another local critical stress and slenderness need to be determined.

5. M-V INTERACTION ASSESSMENT

The applicability of the M-V interaction criterion, given by Eq.(1), in which the new proposed bending and shear resistance models are implemented, is assessed against the numerical results in Fig. 10, for different $M_{f,Rk}/M_{eff,Rk}$ ratios, and curvature pararmeters Z.

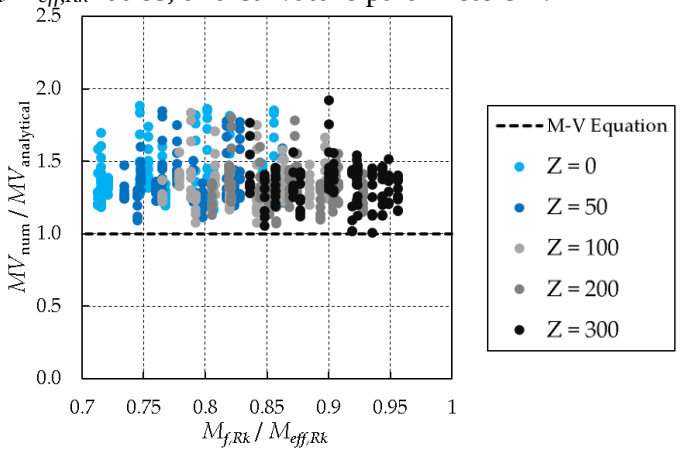


Figure 10 – FEM results vs M-V interaction formula for various $M_{f,Rk}/M_{eff,Rk}$ ratios

The results indicate that none of the points is under the interaction equation when only the web contribution to the shear resistance is considered ($V_{bw,Rk}$). Namely, the minimum differences obtained vary between app.17% and 1.0% on the safe side, from $Z = 0$ to $Z = 300$, respectively, which confirms

that the M-V interaction equation may be safely applied across the entire range of parameters covered in this study ($M_f/M_{eff} \approx 0.7–1.0$).

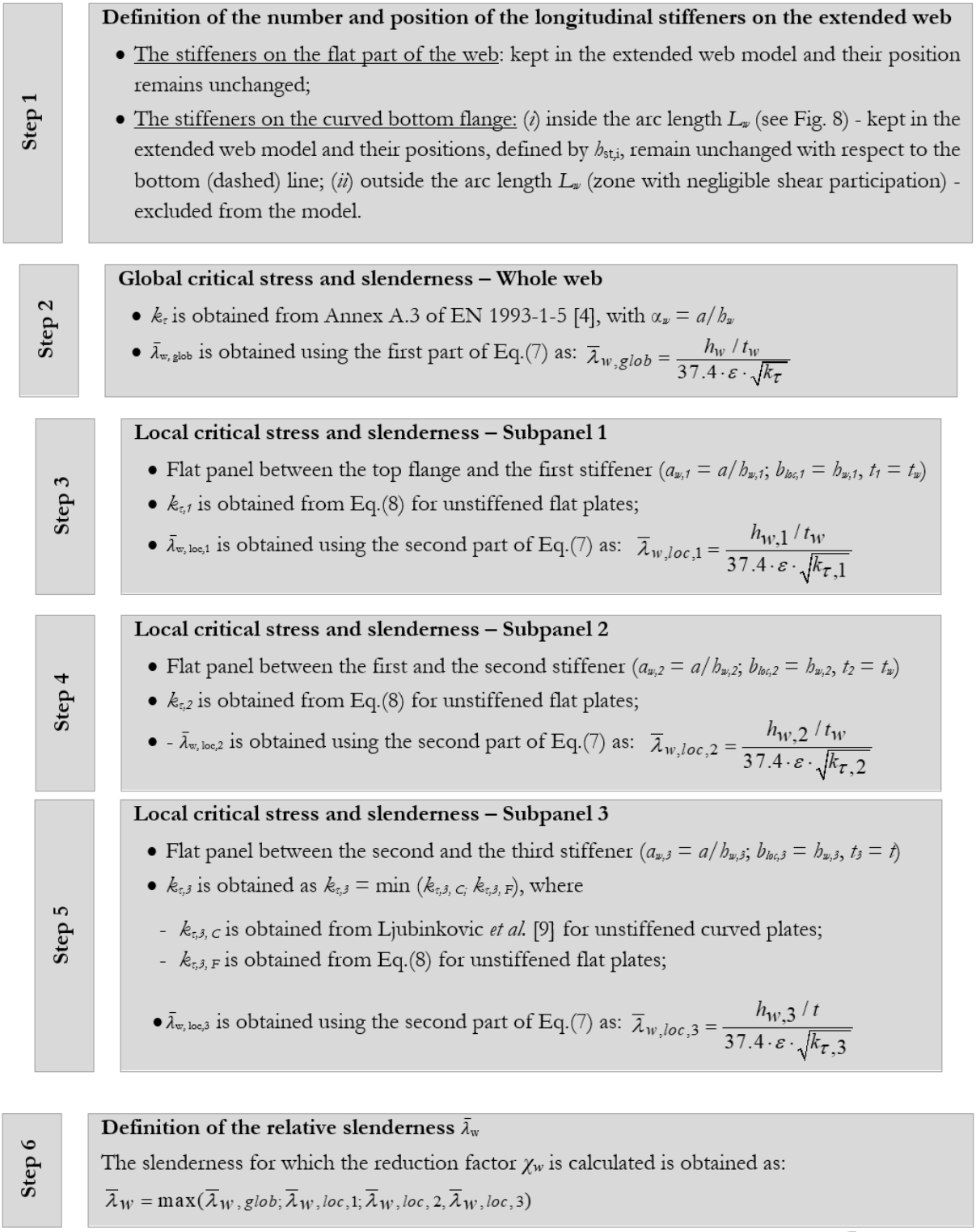


Figure 9 –Flowchart – calculation of the slenderness parameter $\bar{\lambda}_w$

In addition, all three models (bending resistance model, shear resistance model, and the M-V interaction equation), given by Eqs.(1)-(3), are statistically assessed against the numerical results by calculating the partial factor (γ_M). The partial factor γ_M is calculated on the basis of the 4-step approach from SAFEBRICTILE research project [10], which is based on the procedure adopted in Annex E of prEN 1993-1-1 [11]. For the overall assessment, a total of $n = 2100$ results are considered. The results indicate that the equivalent partial factor shows a declining trend with the increase of

curvature, varying approximately from $\gamma_M = 1.06$ for $Z = 0$ to $\gamma_M = 0.97$ for $Z = 300$, depending on the considered subset og geometries.

In conclusion, although the partial factors obtained are slightly lower than those currently recommended in EN 1993-2, the authors suggest to keep $\gamma_{M,0} = 1.0$ and $\gamma_{M,1} = 1.1$

6. CONCLUSIONS

In this paper, two main goals were accomplished. First, the design rules for the assessment of the bending ($M_{eff,Rd}$) and shear ($V_{bw,Rd}$) resistance for the box-girder bridge decks with a curved bottom flange were proposed and statistically assessed. Both models use the design rules for trapezoidal box-girder bridge cross-section from EN 1993-1-5; however, adjusted to account for the presence of the curved bottom flange. Secondly, the M-V interaction capacity of such cross-sections was verified using the criterion adopted in prEN 1993-1-5 [6], in which the two aforementioned design models were explicitly implemented.

Regarding the bending resistance model, named *Curved panel approach*, the numerical results give higher bending resistance for all 210 studied geometries (avg. 30%), which may be attributed to multiple simplifications adopted, such as the assumption that the entire curved bottom flange is subject to the most unfavourable uniform axial stress gradient.

Relatively to the developed shear resistance model, named *Extended web method*, it may be concluded that the model is safe, presenting a lower bound of all numerical results, except for two cases where the FEM results were overestimated for 2% and 6%. Moreover, being an extension of the conservative EN 1993-1-5 approach, the model is in most cases overly safe; however, the difference with respect to the FEM results tends to decrease to a satisfactory 29% with the increase of curvature.

Subsequently, the safety of the M-V interaction equation was evaluated against the FEM results and it was concluded that the M-V interaction equation may be safely applied across the entire range of parameters covered in this thesis ($M_f/M_{eff} \approx 0.7–1.0$), with the minimum differences between numerical and analytical results of app. 17% and 1.0% on the safe side, from $Z = 0$ to $Z = 300$, respectively.

Finally, all three resistance models (*i.e.* bending, shear, and the M-V interaction) were statistically assessed against the FEM results by calculating the partial factor (γ_M). The obtained partial factors were slightly lower than those recommended by EN 1993-2. Hence, it is suggested to keep $\gamma_{M,0} = 1.0$ and $\gamma_{M,1} = 1.1$ in the design of box-girder bridges with a curved bottom flange.

7. ACKNOWLEDGMENTS

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