T. SIMONOVIĆ, M. STAMENIĆ, N. TANASIĆ, M. TRNINIĆ

Impact of incident angle deviation on parabolic trough collector performance

Parabolic-trough solar collectors (PTCs) have emerged as a cost-effective and sustainable solution for solar power generation [1]. While primarily used in power plants, PTCs also have applications in domestic settings, such as hot-water heating. Although typically oriented with a north-south longitudinal axis, east-west configurations are also feasible [1], and other orientations are possible, albeit less common. The longitudinal axis can be positioned horizontally or tilted relative to the horizontal plane.

1 Introduction

Due to the continuous change in the Sun's position (a function of date, time, and collector location (longitude and latitude)), PTCs require constant re-orientation (rotation around the longitudinal axis) to maintain the focal plane parallel to incident solar radiation. Optimal thermal output is achieved when solar beams are perpendicular to the collector's longitudinal axis. Any deviation β (°) from this ideal alignment reduces PTC thermal performance. This effect can be intentionally employed to reduce collector output, for example, for safety reasons. Furthermore, this analysis is relevant to determining the required precision of solar tracking devices and manufacturing tolerances of PTC components.

This analysis aims to determine the minimum β_{min} (°) and maximum β_{max} (°) deviation angles that correspond to negligible and maximal reductions in collector thermal output, respectively. The ultimate goal is to establish a functional relationship between the deviation angle (β) and thermal output within the range [β_{min}, β_{max}].

2 The basic elements and geometrical characteristics of PTC

The basic elements of a PTC are a reflector (1), a linear heat absorber (2), support construction (3), piping (4) (Fig. 1), and a tracking device.

The fundamental geometry of a parabolic trough collector (PTC) is defined by several key parameters: length L (m), width B (m), focal distance f (m), and the diameter d (m) of the linear heat absorber. Figure 2 illustrates the cross-sectional and side views of a PTC, highlighting these important geometrical characteristics.

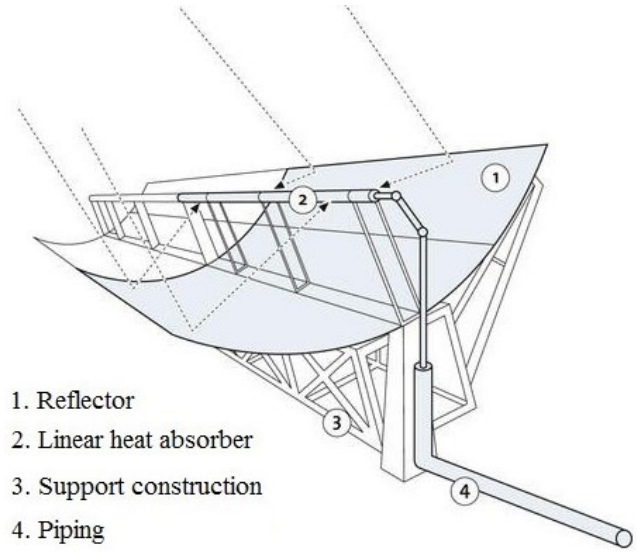

Figure 1: Parabolic-trough solar collectors basic elements [3]

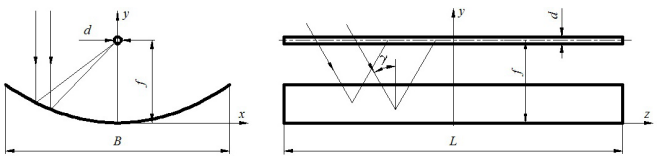

Figure 2: A cross-section and side view of PTC

A coordinate system can be established at the base of the parabolic reflector, as shown in Figure 2. The equation of the parabolic curve can then be expressed as $y = x^2 / (4 \cdot f), x = \{-B/2, B/2\}$, where f is the focal distance (m).

It is necessary to continuously adjust the PTC's y-axis to keep the parabola's focal plane parallel to the incoming sunlight. The PTC can be rotated either around its longitudinal (z) axis or around the linear heat absorber, in which case the coordinate system would be centered at the focal point.

The results of this analysis remain invariant to the specific positioning of the coordinate system. The coordinate system is fixed to the collector and rotates along with it, following the Sun's position.

3 The thermal performance of PTC

Calculating the thermal performance of a Parabolic Trough Collector (PTC) is complex. To simplify the process, stochastic variables such as local weather fluctuations, surface cleanliness, and solar variations are typically neglected. Nevertheless, PTC thermal performance relies on numerous variables. The normalized thermal performance can be expressed using equation (1):

$$I = I_s \cdot \eta_a \cdot \eta_c \cdot \eta_o \cdot \eta_h \cdot \eta_\beta \ , \text{W/m}^2 \qquad (1)$$

Where:

$$I_s = 1367 \cdot \left[1 + 0.033 \cdot \cos\left(360 \cdot n / 365\right)\right] \ , \text{W/m}^2$$

This represents the solar radiation power at Earth's distance from the Sun, but outside the atmosphere. It is a periodic function of Earth's revolution, depending on the specific day of the year, ranging from 1 to 365 [4].

η_a: This coefficient accounts for atmospheric absorptivity. It varies based on atmospheric composition, location, time of day, and time of year. It can be determined analytically using specific assumptions or measured more precisely on-site [4].

$\eta_c = \cos\gamma$: This cosine factor depends on the relative angular position of the collector and the direction of the sunbeams. Here, γ represents the angle between the collector's y-axis and the incident sunbeams [5], as illustrated in Figure 2.

η_o: The total optical efficiency encompasses all losses related to the collector's optical properties. It includes losses at the collector ends, mirror reflectivity, glass transparency, absorber absorptivity, manufacturing imperfections in the collector and tracking device, and positioning errors. It is the most complex coefficient because optical efficiency varies across the collector surface; the total efficiency is calculated by integrating over the entire surface [5].

η_h: The heat loss coefficient accounts for thermal losses from the linear heat absorber via convection, conduction, and radiation [5].

η_β: The angle deviation coefficient reflects the decrease in thermal performance caused by small deviations of the PTC's focal plane relative to the direction of incident sunbeams (dependent on β).

The methodology for calculating η_c, η_o, and η_h, along with essential data such as the reflectivity, transparency, and absorptivity of common materials, is detailed in [5].

4 Analysis

Generally, Parabolic Trough Collectors (PTC) rotate solely around their longitudinal axis (z-axis). While other movements are possible, they are rarely implemented. In contrast, the Sun's apparent position is defined by two angles (azimuth and altitude), both of which depend on time and the PTC's specific location. Consequently, the calculation of the coefficients in equation (1) is a three-dimensional problem, and except η_β, all coefficients are time- and location-dependent.

Assumptions and simplifications

The angle deviation coefficient (η_β) can be determined by treating it as a two-dimensional problem, since losses related to the third dimension are already accounted for in the coefficients η_c and η_o. This analysis focuses only on small-angle deviations between the incident beam and the focal plane ($\beta < 2$). Under these conditions, the influence of angle deviation on coefficients other than η_β is negligible.

Several other simplifications are made:

- **Solar Source**: The Sun is not a point source; sunbeams are not perfectly parallel and deviate by approximately 32' [4]. This imperfection is neglected in this paper.
- **Beam Loss**: Any beam that misses the linear heat absorber is considered a total loss. The presence of radiation shields, typically installed to reflect radiation to the absorber, is ignored.
- **Surface Quality**: Since coefficient η_o accounts for collector imperfections, η_β is calculated assuming the parabolic surface is perfectly smooth.
- **Refraction**: The influence of beam refraction through the glass tube (which surrounds the absorber to reduce convective heat loss) is neglected.

Coefficient of angle deviation η_β

Treating this analysis as a two-dimensional problem significantly simplifies the calculation of the angle deviation coefficient (η_β) by allowing the use of planar analytical geometry.

Figure 3 illustrates a PTC cross-section where the focal plane (y-axis) is tilted away from the direction of the sunbeams by a small angle β. Sunbeams striking the collector at coordinate ($-x_1$) reflect and hit the linear heat absorber, whereas beams striking at coordinate ($-x_2$) reflect and miss the absorber.

It can be demonstrated that at a minimal angle deviation β_{min}), all reflected beams still hit the absorber, meaning the entire PTC surface remains useful. However, as the deviation angle increases beyond β_{min}, the useful surface area decreases because beams reflected from the left and right ends of the PTC begin to miss the target. When the deviation angle reaches β_{max}, all reflected beams miss the absorber, resulting in zero useful PTC surface.

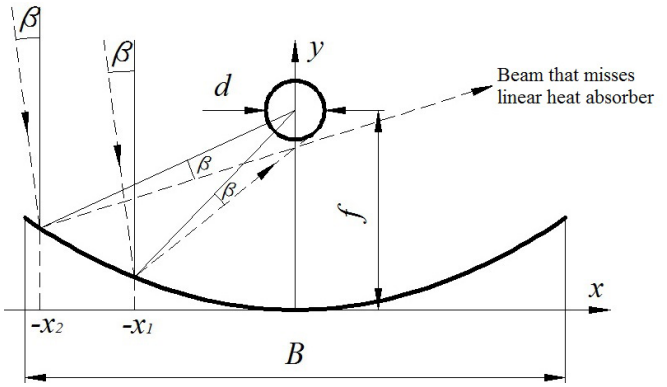

Figure 3: Small deviation of the incident angle of the Sunbeam at a parabolic-trough solar collector

If the point with coordinate ($-x_1$) represents the location where a reflected beam is exactly tangential to the linear heat absorber, a relationship exists between the angle β and coordinate ($-x_1$), as shown in equation (2). In this context, ($-x_1$) serves as a critical boundary. If $|x| > |x_1|$, the beam reflected from coordinate x will miss the absorber; otherwise, it will strike it. Given the symmetry of the PTC, the surface area between coordinates $-x_1$ and x_1 defines the useful area, as all beams reflected within this interval successfully reach the linear heat absorber.

$$\sqrt{x_1^2 + (f - y_1)^2} \cdot \sin\beta = \frac{d}{2} \tag{2}$$

By substituting the analytical expression of the parabola into equation (2), we derive equation (3), which relates the coordinates of the useful area to the angle β.

$$x_1 = \pm 2 \cdot f \sqrt{\frac{d}{2 \cdot f \cdot \sin\beta} - 1} \tag{3}$$

The useful area $A_u = 2 \cdot x_1 \cdot L \cdot \sin\beta$ and the total area $A = B \cdot L \cdot \sin\beta$ of the PTC are calculated in square meters (m²).

The angle deviation coefficient, η_β, can then be determined using equation (4).

$$\eta_\beta = \frac{A_u}{A} = \frac{2 \cdot x_1 \cdot L \cdot \sin\beta}{B \cdot L \cdot \sin\beta} = \frac{2 \cdot x_1}{B} = \frac{4 \cdot f}{B} \cdot \sqrt{\frac{d}{2 \cdot f \cdot \sin\beta} - 1} \tag{4}$$

Equation (4) provides an analytical relationship between η_β and the deviation angle β for a specific collector geometry. However, this equation is valid only when the angle deviation β falls within the interval (β_{min}; β_{max}). Therefore, the coefficient η_β is more precisely expressed using equation (5).

$$\eta_\beta = \begin{cases} 1, & \beta \le \beta_{\min} \\ \dfrac{4 \cdot f}{B} \cdot \sqrt{\dfrac{d}{2 \cdot f \cdot \sin\beta} - 1}, & \beta_{\min} \le \beta \le \beta_{\max} \\ 0, & \beta \le \beta_{\max} \end{cases} \tag{5}$$

The minimal deviation angle (β_{min}) - the threshold below which there is no reduction in thermal output, can be determined using equation (3). By setting the coordinate x_1 equal to the collector half-width ($B/2$), the entire PTC area remains useful, leading to equation (6).

$$\beta_{\min} = \sin^{-1}\left[\frac{8 \cdot d \cdot f}{16 \cdot f^2 + B^2}\right] \tag{6}$$

Conversely, the maximal deviation angle, β_{max}, is determined using equation (3) by setting the coordinate x_1 to 0. At this point, the PTC has no useful area, yielding equation (7).

$$\beta_{\max} = \sin^{-1}\left[\frac{d}{2 \cdot f}\right] \tag{7}$$

5 Simulation on real PTC-type LS-1

Reference [1] details the fundamental characteristics of several PTC types commonly used in power generation. For this analysis, the LS-1 collector, used in California's SEGS I and II power plants, was selected. Its primary geometrical specifications are defined as follows: width B = 2.55 m, focal distance f = 0.94 m, length L = 50.2 m, and absorber diameter d = 0.04 m [1].

Using these parameters in conjunction with Equations (6) and (7), the minimal and maximal deviation angles are calculated to be β_{min} = 0.835 = 50.1' and β_{max} = 1.219 = 73', respectively. Figure 4 illustrates the relationship between the deviation angle (β) and the angle deviation coefficient (η_β) for the LS-1 collector.

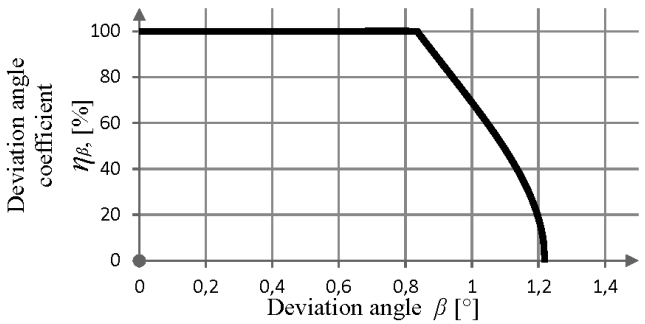

Figure 4: Relation between deviation angle β and deviation angle coefficient η_β for PTC LS-1

When the deviation angle β exceeds 1.2° the coefficient η_β drops to zero. This sensitivity is a consequence of the linear heat absorber's small diameter (d = 0.04 m) relative to the overall collector geometry, particularly its width (B = 2.55 m). The LS-1 collector can use such a small diameter due to its high manufacturing precision. The absorber diameter significantly influences the angle deviation coefficient; for instance, increasing the diameter to d = 0.1 m shifts the minimum and maximum deviation angles to β_{min} = 2.088° and β_{max} = 3.049°, respectively.

6 Conclusions

The deviation angle β significantly influences PTC thermal output; even minor deviations beyond β_{min} can lead to a substantial reduction. This analysis, which is purely geometrical and based on several assumptions and simplifications, means that the derived relationship between the deviation angle and its corresponding coefficient (Equation 5) may differ from that observed in real-world PTC installations. In actual systems, it is expected that the coefficient η_β will begin to decrease immediately for any $\beta > 0$, with a more pronounced drop occurring when $\beta > \beta_{min}$.

The diameter of the linear heat absorber (d) profoundly impacts both, β_{min} and β_{max}, as well as the functional relationship between the deviation angle and its coefficient. Opting for a larger absorber diameter will decrease the concentration ratio but concurrently

enhance operational flexibility.

While the Sun's apparent position shifts continuously due to Earth's rotation, the PTC does not require constant tracking, allowing periodic adjustments. The minimal deviation angle β_{min}) can be utilized to determine an appropriate correction time period for these periodic movements.

References

[1] **Price H., E. Lüpfert, D. Kearney, E. Zarza, G. Cohen, R. Gee, R. Mahoney**, *Advances in Parabolic Trough Solar Power Technology*, Journal of Solar Energy Engineering, vol. 124, pp. 109-125, 2002.

[2] **Arasu A. V., T. Sornakumar**, *Design, manufacture and testing of fiberglass reinforced parabola trough for parabolic trough solar collectors*, Solar Energy, vol. 81, pp. 1273–1279, 2007.

[3] Internet presentation: http://www.solarmillennium.de/english/archives/technology/parabolic-trough-power-plants/solar-field/index.html

[4] **Duffie J. A., W. A. Beckman**, *Solar Engineering of thermal processes*, Second Edition, John Wiley and Sons, New York, USA, 1980.

[5] **Huang W., P. Hu, Z. Chen**, *Performance simulation of a parabolic trough solar collector*, Solar Energy, vol. 86, pp. 746–755, 2012.

Authors

Tomislav SIMONOVIĆ
The Academy of Applied Studies Polytechnic, Belgrade, Serbia

Mirjana STAMENIĆ
Department for Process Engineering, University of Belgrade, Faculty of Mechanical Engineering,

Nikola TANASIĆ
The Academy of Applied Studies Polytechnic, Belgrade, Serbia

Marta TRNINIĆ
The Academy of Applied Studies Polytechnic, Belgrade, Serbia